Analysis of multibody dynamics under uncertainty using Time-dependent polynomial chaos

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EXTENDED ABSTRACT

1 Abstract

The uncertainty of mechanical systems can occur from a lack of information such as uncertain system parameters, random inputs, random initial conditions, and random forces. In deterministic analysis, However, these uncertainties can not be considered because uncertain variables are often replaced by just deterministic mean values. To consider these uncertainties, statistical approaches are commonly adopted. The Monte Carlo method is the most popular approach to account for uncertainty. The Monte Carlo method can be easily applied to any problems and guarantee high accuracy regardless of the nonlinearity and complexity of the problems. However, the Monte Carlo method is very expensive due to its slow convergence in terms of computational cost. It is often impossible to use the Monte Carlo method in large mechanical systems. Recently, the polynomial chaos method is getting attention because of its fast convergence property and ability to describe the uncertainty with functional representation. Polynomial chaos has been widely developed in structural engineering and fluid engineering fields. The first application of the polynomial chaos in multibody dynamics is carried out by Sandu et al. [1]. Sandu [1] successfully applied the polynomial chaos method to constrained multibody dynamics systems. The polynomial chaos method can provide the stochastic representation of the response of multibody dynamics systems in both time and frequency domains. However, it encounters a problem in the case of the time domain with long-time integration. Generally, the orthogonal polynomial basis has optimality for the distribution of the random input at the initial time, but after long-time integration, the basis loses its optimality [2]. In the sufficiently small initial time interval, output random distribution can be represented by a combination of only a few terms of low order basis that is optimally decided at initial. However, over time integration proceeds, the distribution of output is diversed into a unique distribution, so higher order terms of basis are additionally required to represent the distribution. In [2, 3], a basis update method for arbitrary measures is proposed to solve the long-time integration problem. Gerritsma [2] propose the time dependent polynomiacl chaos(TD-PC) process with the Gram-Schmidt orthogonalization to reconstruct optimal basis with arbitrary measures in the first order ordinary differential equation. Ozan [3] applied the Karhunen-Loeve expansion into the basis update process for non-Gaussian random process output. In this work, we apply the time dependent polynomial chaos in the multibody dynamics problem to investigate the statistical uncertainty of the response after a long-time integration with only a few terms of low order basis.
In practical engineering systems, physical responses under uncertainty can be represented as a random process as

$$u(t,\omega) = \sum_{\boldsymbol{\beta}}^{\infty} u_{\boldsymbol{\beta}}(t)\phi_{\boldsymbol{\beta}}(\boldsymbol{\xi}(\omega)), \tag{1}$$

in terms of time and random domain. Here $\phi_{\boldsymbol{\beta}} = \phi_{\beta_1} \times \cdots \times \phi_{\beta_d}$ are generalized Askey-Wiener scheme orthogonal polynomials, $\boldsymbol{\xi} = (\xi_1, \cdots, \xi_d)$ is the finite number of random variable for the random event ω and $\boldsymbol{\beta} = (\beta_1, ..., \beta_d)$ is the multi-indices. This linear combination of mutually orthogonal polynomial basis converges in L^2 sense [4]. The coefficients of polynomial chaos expansion can be obtained by the orthogonal characteristic of polynomials as

$$u_{\boldsymbol{\beta}}(t) = \frac{1}{\parallel \phi_{\boldsymbol{\beta}}(\boldsymbol{\xi}) \parallel} \int u(t,\omega)\phi_{\boldsymbol{\beta}}(\boldsymbol{\xi})f(\boldsymbol{\xi})d\boldsymbol{\xi}, \quad \parallel \phi_{\boldsymbol{\beta}} \parallel = \int \phi_{\boldsymbol{\beta}}^2(\boldsymbol{\xi})f(\boldsymbol{\xi})d\boldsymbol{\xi}, \tag{2}$$

with respect to the joint probability density $f(\boldsymbol{\xi})$ of the random variable $\boldsymbol{\xi}$. In practice, equation (1) is truncated in a finite number of basis $N = (d+P)!/(d!P!)$, where d is the stochastic dimension of the random variable and P is the order of the polynomial chaos. The finite number of polynomial chaos expansion is represented as

$$u(t,\omega) = \sum_{|\boldsymbol{\beta}| \leq P} u_{\boldsymbol{\beta}}(t) \prod_{j=1}^{d} \phi_{\beta_j}(\xi_j(\omega)). \tag{3}$$

The initially defined orthogonal basis is a function of the input random variable and depends only on the distribution of the random variable. To consider the distribution of a uniquely divergent random variable at time t_i, arbitrary polynomial basis has to be constructed. Here the time domain is decomposed into n interval as $\{0,t_1\}, \{t_1,t_2\}, ..., \{t_{n-1},t_n\}$. The arbitrary polynomial basis defined as

$$\left\{ \phi_{\boldsymbol{\beta}}^{(t_i)}(\boldsymbol{\xi}, u^{(t_i)}); \quad | \boldsymbol{\beta} | \leq P, \quad \boldsymbol{\beta} = (\beta_1, \cdots, \beta_{d+m}) \right\}, \tag{4}$$

where $u^{(t_i)}$ denotes the system output at time t_i and m is the number of output variables. The number of total basis at time t_i is $N^{(t_i)} = ((d+m)+P)!/((d+m)!P!)$. Equation (4) is constructed based on the first $2N^{(t_i)}$ multivariate moments of random variables. The Gram matrix can be represented as

$$[G_{i,j}] = \int (\boldsymbol{\xi} \times u^{(t_i)})^{\boldsymbol{\beta}_i} (\boldsymbol{\xi} \times u^{(t_i)})^{\boldsymbol{\beta}_j} d\mathscr{P}, \quad i,j = 1,\cdots,N^{(t_i)}, \tag{5}$$

where $\mathscr{P}$ is a probability measure and the elements are multivariate moments up to $2N^{(t_i)}$. Upper triangular matrix R can be obtained by the Cholesky factorization $G = R^T R$. From the Gram matrix, the orthogonal polynomial basis can be obtained as

$$\phi_{\boldsymbol{\beta}_j}^{(t_i)} = r_{1,j}(\boldsymbol{\xi} \times u^{(t_i)})^{\boldsymbol{\beta}_1} + \cdots + r_{j,j}(\boldsymbol{\xi} \times u^{(t_i)})^{\boldsymbol{\beta}_j}. \tag{6}$$

where r are the elements of invers matrix R^{-1} of upper triangular matrix. For each time interval, a new orthogonal polynomial basis is constructed based on the response of t_i obtained in the previous time interval. The implementation of a time dependent polynomial to a multibody dynamics is demonstrated using the single pendulum system. The multibody dynamics formulation of the single pendulum is as follows

$$\begin{bmatrix} m & 0 & 0 & 1 & 0 \\ 0 & m & 0 & 1 & 1 \\ 0 & 0 & J & L\sin\theta & -L\cos\theta \\ 1 & 0 & L\sin\theta & 0 & 0 \\ 0 & 1 & -L\cos\theta & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -mg \\ 0 \\ -L\dot{\theta}^2\cos\theta \\ -L\dot{\theta}^2\sin\theta \end{bmatrix}, \tag{7}$$

where $L = \bar{L} + L^{'}\xi(\bar{L} = 1m, L^{'} = 0.15m)$ is the random length of the pendulum, θ is the angular position, λ is the Lagrange multiplier, and m, J, g are the mass of the pendulum, the moment of inertia, and the gravitational acceleration, respectively. The random variable ξ is a normalized uniform distribution with zero mean and variance 1, and a Legendre polynomial is employed for the initial polynomial basis. To apply the Galerkin projection to the equation (7), trigonometric terms are substituted to linear terms as $L\sin\theta = x$, $L\cos\theta = y$, where x and y are Cartesian coordinates of the pendulum. Figure 1 shows the time evolution of multibody dynamics system under uncertainty. A Monte Carlo method was performed to compare the results of time dependent polynomial chaos solutions. In this study, we applied the time dependent polynomial chaos in the multibody dynamics system. Time dependent polynomial chaos seems to be able to obtain the stochastic representation of multibody dynamics response over long-time integration with only a few terms of low order basis.

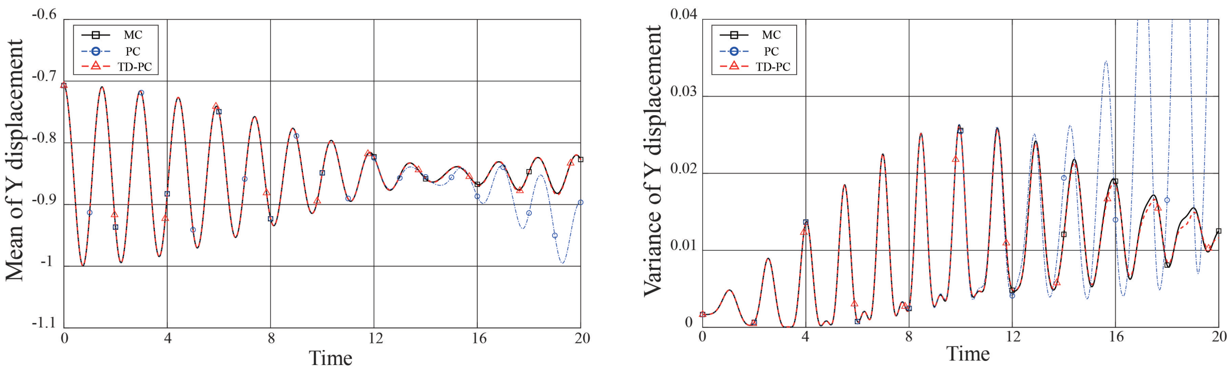

Figure 1: The mean and variance of single pendulum result with polynomial chaos order P = 3 and Monte Carlo sample size = 100,000

References

[1] Sandu, Adrian, Corina Sandu, and Mehdi Ahmadian. "Modeling multibody systems with uncertainties. Part I: Theoretical and computational aspects." Multibody System Dynamics 15.4 (2006): 369-391.

[2] Gerritsma, Marc, et al. "Time-dependent generalized polynomial chaos." Journal of Computational Physics 229.22 (2010): 8333-8363.

[3] Ozen, H. Cagan, and Guillaume Bal. "A dynamical polynomial chaos approach for long-time evolution of SPDEs." Journal of Computational Physics 343 (2017): 300-323.

[4] Xiu, Dongbin. "Numerical methods for stochastic computations." Numerical Methods for Stochastic Computations. Princeton university press, 2010.