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Design Optimization of Multi-Elastic-Link Robots Based on Representative Load Cases

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EXTENDED ABSTRACT

1 Introduction and Problem Description

This paper addresses the design optimization of serial robotic manipulators with slender arms possessing significant structural elasticities, based on the dynamic simulations for representative cyclic trajectories. The resulting stresses are used for a fatigue analysis and subsequent lifetime estimation of the critical structural elements. This is then used for optimizing the robot geometries, e. g. cross sections or centers of gravity.

2 Modeling

The considered articulated robot is build up of two different types of subsystems: a rotating base and elastic arms with motor-gearbox units which are assumed to be elastic as well. The arms shall fulfill the requirements of the Euler-Bernoulli theory and undergoing multiaxial bending and torsion around the beam axis (see Fig.1). As stated in [1], the displacement of the beam axis is described by $v(\xi,t) = \mathbf{v}(\xi)^T \mathbf{q}_v(t)$ and $w(\xi,t) = \mathbf{w}(\xi)^T \mathbf{q}_w(t)$ while its torsion angle is $\vartheta(\xi,t) = \boldsymbol{\vartheta}(\xi)^T \mathbf{q}_\vartheta(t)$. $\xi \in \{0, L\}$ denotes the coordinate alongside the beam axis. $\mathbf{v}(\xi)$, $\mathbf{w}(\xi)$ and $\boldsymbol{\vartheta}(\xi)$ denote the shape functions. $\boldsymbol{q}_e = \left(\boldsymbol{q}_v^T, \boldsymbol{q}_w^T, \boldsymbol{q}_\vartheta^T\right)^T$ are the time dependent Ritz coefficients. Furthermore, the motor angle $q_M(t)$ and the arm angle $q_A(t)$ are introduced. $\boldsymbol{v}_0^T$ and $\boldsymbol{\omega}_F^T$ describe the movement of the coupling frame between the link and its predecessor. In order to account to the complexity of the full elastic dynamic model the recursive subsystem $\mathcal{O}(n)$-Algorithm as stated in [1] is used. This allows for representing subsystems in different detailing, as required for the investigation of the sources of vibrations.

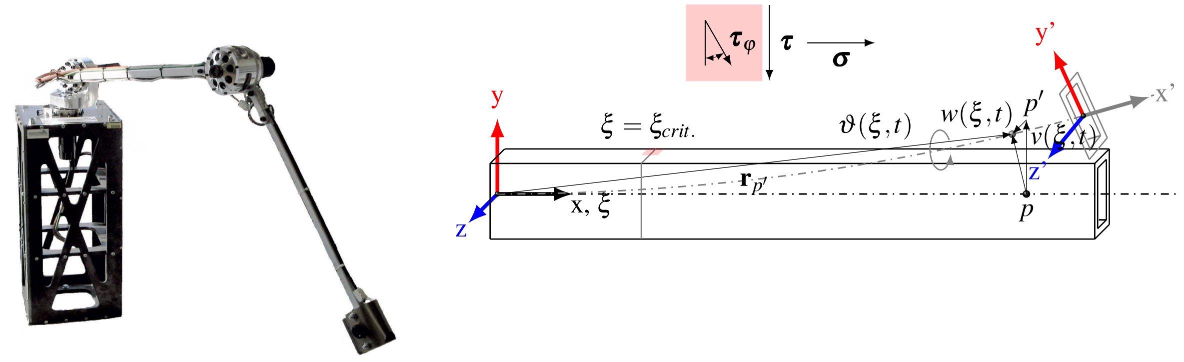

Figure 1: Elastic robot ELLA (left) and elastic link (beam) of an articulated robot, with elastic displacements and critical point for the subsequent lifetime estimation (right)

3 Multicriterial Optimization of the Robot Structure

Structure optimization encompasses the variation of selected parameters $\tilde{\boldsymbol{p}}$ such as beam geometries, center of gravity (COG) coordinates, etc., through adjustments of the robots mechanical structure. The goal to design the robot as light but also as stiff as possible, gives rise to the following, multicriterial, optimization problem

$$\min_{\tilde{\boldsymbol{p}} \in \boldsymbol{P}} \left(J_{\mathrm{m}}\left(\tilde{\boldsymbol{p}}\right), J_{\mathrm{osc.}}\left(\tilde{\boldsymbol{p}}, \boldsymbol{q}(t)\right)\right). \tag{1}$$

$\boldsymbol{P}$ denotes the set of valid parameters and $\boldsymbol{q}(t)$ is the considered trajectory in the joint space. $J_{\mathrm{m}}\left(\tilde{\boldsymbol{p}}\right) = 100\frac{m(\tilde{\boldsymbol{p}})}{m^0}$ is the weight in percent with respect to an initial robot configuration with mass m^0. $J_{\mathrm{osc}}\left(\tilde{\boldsymbol{p}}, \boldsymbol{q}(t)\right)$ denotes a quality criteria regarding the oscillation of the endeffector E. As one can focus on the position error $\Delta_I \boldsymbol{r}_E\left(t, \tilde{\boldsymbol{p}}\right) = {}_I\boldsymbol{r}_{E,d}(t) - {}_I\boldsymbol{r}_E\left(q_M(t), q_A(t), \boldsymbol{q}_e\left(t, \tilde{\boldsymbol{p}}\right)\right)$ or the orientation error $\Delta_I \boldsymbol{\varphi}_E\left(t, \tilde{\boldsymbol{p}}\right) = {}_I\boldsymbol{\varphi}_{E,d}(t) - {}_I\boldsymbol{\varphi}_E\left(q_M(t), q_A(t), \boldsymbol{q}_e\left(t, \tilde{\boldsymbol{p}}\right)\right)$, where ${}_I\boldsymbol{r}_{E,d}(t)$ and ${}_I\boldsymbol{\varphi}_{E,d}(t)$ denote the desired position/orientation, there are several possible cost functions, as the errors whole time evolution, the maximum error or also the mean squared error $\frac{1}{T}\int\limits_{t=t_0}^{t=t_0+T} \Delta_I \boldsymbol{r}_E dt$ over an interesting time window.

4 Lifetime Analysis of the Optimized Robot

As the Ritz coefficients $\mathbf{q}_e$ are accessible as a result of the dynamic simulation, it is possible to determine the elastic displacement of every point $\xi \in \{0, L\}$ on the beam axis. The corresponding normal and shear stresses are $\sigma(\xi, t)$ and $\tau(\xi, t)$. For homogenous beams with constant cross section, the weakest part of the mechanical structure tends to be the connection of the joints and the links, so this is the point of interest $\xi_{\text{crit.}}$. The proposed lifetime estimation, see algorithm 1, is based on a fusion of the method of the critical cutting plane and the rainflow method, with the cumulative damage equations according to Palmgren-Miner, see e. g. [2] for all three methods.

Algorithm 1: Lifetime estimation of a critical point in the robot's structure on basis of the method of the *Critical Cutting Plane*. For converting the uniaxial loading sequence into an equivalent set of constant amplitude stress reversals, the *Rainflow Counting Algorithm* is used. The method assumes linear damage accumulation according to *Palmgren-Miner*.

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Input: Set of cutting angles $\Phi$, material specific Haigh diagram
Data: Normal and shear stresses $\sigma(\xi,t)$ and $\tau(\xi,t)$ at $\xi = \xi_{\text{crit.}}$
for $\varphi \in \Phi$ do
    calculate shear stress $\tau_\varphi(\xi,t) = -\frac{1}{2}\sigma(\xi,t)\sin(2\varphi) + \tau(\xi,t)\cos(2\varphi)$ for the $\varphi$ cutting plane and $t = [0, t_{\text{sim}}]$
    calculate rainflow matrix $\boldsymbol{M}(\tau_m, \tau_a)$ with the numbers $N(\tau_m, \tau_a)$ of reversals of the stress amplitudes with mean stress $\tau_m$
      and stress amplitude $\tau_a$
    for $(\tau_m^*, \tau_a^*) \in \boldsymbol{M}$ do
        begin calculate the related damage growth $\Delta D$
            equivalent normal stresses $\sigma_m^* = 2\tau_m^*$ and $\sigma_a^* = 2\tau_a^*$ according to the maximum shear stress hypothesis
            fatigue strength $\sigma_{D,a}^*$ for $\sigma_m^*$ from Haigh diagram
            damage $\Delta D = N(\tau_m^*, \tau_a^*) / N(\sigma_a)$ with $N(\sigma_a)$ from the synthetic Wöhler line for $\sigma_m^*$
        end
        $D(\varphi) := D(\varphi) + \Delta D$ according to the Palmgren-Miner damage hypothesis, $D \in [0,1]$ represents the damage, $D = 1$
          means failure of the considered component
    end
    $D_{\max} := \max(D(\varphi), D_{\max})$
end
Result: Estimated lifetime $t_{\text{life}} := t_{\text{sim}} / D_{\max}$
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5 Conclusion and Outlook

The introduced procedure provides a way to design and verify a lightweight robot with a (within the constructive possibilities) tailored dynamic behavior under assumption of a representative task trajectory. Applying it to an industrial pick and place robot, the multicriterial structure optimization resulted in a pareto front with respect to mass reduction and endeffector oscillation, where each point corresponds to parameters $\tilde{\boldsymbol{p}}$. The introduced lifetime estimation then provided a criterion for choosing the final design. An estimation of the remaining lifetime could be realized by utilizing the shown method for the damage calculation of the elastic links to accumulate the damage over the robots operation period to date. In addition to what we show in this work further weight reduction at the joints (drive units, etc.) might be quantified by additional elasticities, where the stiffness is determined by FEM simulations of the whole joint structure, as proposed in [3].

Acknowledgments

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References

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