# Tire cornering stiffness estimation using hybrid modeling

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### 1 Introduction

In vehicles, tires have a profound impact on vehicle dynamics including their handling, drivability and overall comfort. Advanced chassis control systems being used in vehicles to improve their performance, reliability, and safety also require knowledge of vehicle parameters, e.g., tire cornering stiffness to evaluate lateral forces [1, 2]. There also is a lack of detailed expert knowledge on the specifics, i.e., the internal structure and material properties of tires due to intellectual property and the wide variety of tires. Although simplified models are sometimes employed to simulate tires, such models might be unable to capture the complexity and nonlinearity of tire behaviors. In addition, techniques based on physical vehicle models, in which tire loads are considered, inherently include uncertainty due to changes in the loading and tire properties. When a rolling tire is subjected to both vertical and lateral forces, its motion direction on the road makes an angle, called sideslip angle, with respect to the tire orientation. The lateral force can be considered equal to this angle multiplied by the cornering stiffness as a coefficient [3]. Several factors influence cornering stiffness of tires, namely vertical tire loadings, inflation pressure, tire wear, temperature, and aging, which alter owing to driving, operating, and environmental conditions [4, 5]. Therefore, robust methods to estimate cornering stiffness are required to develop such that they do not need the manufacturer data and physical knowledge on the tire and installing new and expensive sensors, i.e., wheel force transducers. Moreover, they should update themselves automatically based on the environmental conditions and operation. Neural network-based methods can be helpful predicting vehicle parameters without any knowledge of mechanical properties and complex mechanisms using regression methods [4]. However, such approaches do not work well outside the training data distribution. Consequently, new data are to be collected to apply the estimator on different vehicles with components of unlike size and property, resulting in huge costs and extensive time efforts. Thus, this article develops a hybrid model by combining partially known physics of vehicle dynamics and recurrent neural networks, which compensates for the unmodeled physics of tires, to identify cornering stiffness. This approach learns the system automatically according to the data collected from vehicle responses, without requiring costly measured tire forces but simply relying on signals from an IMU (Inertial Measuring Unit) and can subsequently update itself against emerging changes due to environmental conditions, driving and operating. The hybrid model is trained using input data and available system states, along with an error defined on the output of the dynamic model. As the learning process becomes an inherent characteristic of this model while using the data measured by means of the vehicle sensors, one can utilize this hybrid simulation for different vehicle variants and tire dimensions. The proposed methodology is trained using experimental data from track testing of a Siemens SimRod. The wheel cornering stiffnesses are estimated, and the acquired vehicle dynamics are compared to measurements.



Fig. 1. The workflow of the hybrid model developed for the estimation of cornering stiffness of the vehicle.

#### 2 Mathematical modeling

The workflow of the developed hybrid model is presented in Fig. 1. In order to simulate the vehicle motion, two-wheel rigid vehicle dynamics (bicycle model) is considered, and the following linearized equations are obtained [5].

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \quad \mathbf{x} = [v_y, r], \quad \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}u, \quad \mathbf{y} = [a_y, r, \beta]$$
(1)

where

$$\mathbf{A} = -\begin{bmatrix} (C_f + C_r)/mv_x & V_x + (l_f C_f - l_r C_r)/mv_x \\ (l_f C_f - l_r C_r)/(l_{zz} v_x) & (C_f l_f^2 + C_r l_r^2)/(l_{zz} v_x) \end{bmatrix}, \\ \mathbf{B} = \begin{bmatrix} \frac{C_f}{m} \\ \frac{l_f C_f}{l_{zz}} \end{bmatrix}, \\ \mathbf{D} = \begin{bmatrix} \frac{C_f}{m} \\ 0 \\ 0 \end{bmatrix}, \\ \mathbf{C} = -\begin{bmatrix} (C_f + C_r)/(mv_x) & (l_f C_f - l_r C_r)/(mv_x) \\ 0 & -1 \\ 1/v_x & 0 \end{bmatrix}$$
(2)

in which  $v_y$  is lateral velocity,  $a_y$  lateral acceleration, r yaw rate, and  $\beta$  side slip angle. In addition, the system input, u, is steering wheel angle, SWA, the longitudinal speed is depicted by  $v_x$ . m and  $I_{zz}$  are, respectively, the car mass and yaw moment of inertia. The front and rear positions of center of gravity are  $l_f$  and  $l_r$ , respectively. Finally,  $C_f$  and  $C_r$  are the front and rear cornering stiffnesses. In the hybrid model,  $C_f$  and  $C_r$ , are functions of { $\mathbf{u}, \mathbf{x}$ } and can be written as  $C_f(\mathbf{x}, u)$  and  $C_r(\mathbf{x}, u)$ . These functions are unknown and approximated by recurrent neural networks (RNN). Correspondingly, two RNNs are designed including three hidden layers with 25, 10, and 5 neurons and the network input consists of { $v_x$ , SWA,  $a_y$ , r,  $\beta$ ,  $v_y$ }. Tangent hyperbolic, linear, and the rectified linear unit functions are used as activation functions in both networks. The cost function for the hybrid model is also calculated using the root mean sum-of-squared error (RMSSE) measure, Eq. (3), and back-propagation technique is utilized to obtain the vector  $\mathbf{0}$  that encompasses the network weights and biases with a size depending on the architecture of the RNN. To implement the gradient decent method for the backpropagation, the automatic differentiation cannot be used as the automobile physics is not traceable, and three-point differentiation approach is thus employed.

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}}(RMSSE(\boldsymbol{\theta})), \qquad RMSSE = \sqrt{\frac{1}{LN}\sum_{i=1}^{L-N}\sum_{k=i+1}^{i+N}\mathbf{e}_{k,i}^T(\boldsymbol{\theta})\mathbf{e}_{k,i}(\boldsymbol{\theta})}, \qquad \mathbf{e} = \mathbf{y} - \hat{\mathbf{y}}$$
(3)

## 3 Results and discussion

In order to validate the model, the dynamics of a vehicle using a two-wheel rigid vehicle model are obtained with fixed cornering stiffnesses values. The developed hybrid model is firstly applied to that generated dataset and verified. Then, the cornering stiffnesses of front and rear tires are, in turn, estimated using experimental data from track testing of a Siemens SimRod. Specifically, a step maneuver is employed where steering wheel angle varies in the range of [0, 0.8] radian, the learning process stops as the computation converges, Fig. 2d. The vehicle motion obtained, using the estimated values of cornering stiffness of both tires, Figs. 2a and 2b, complies well with the measured data, namely lateral velocity in Figs. 2d (right), as well as lateral acceleration shown in Fig. 2c. The lateral forces are also calculated by multiplying cornering stiffness and respective tire sideslip angle, and subsequently presented in Figs. 2a and 2b (right) in comparison with those from measurements. One can conclude that the outcomes comply well with measured data of the automobile motion. Overall, the capability of the developed method to estimate vehicle parameters and tire non-linear dynamics without costly and hard-to-install instrumentations is demonstrated. The future research direction is to employ a sparse data-driven discovery method to identify the interpretable tire mode [6].



Fig. 2. Left plot: (a), (b) front and rear wheel cornering stiffnesses; (c) lateral acceleration; (d) root mean sum of squared error. Right plot: (a), (b) front and rear lateral forces; (c) Steering wheel angle; (d) lateral velocity.

#### References

- C.L. Pereira, R. Neto, B. Loiola. Cornering stiffness estimation using Levenberg–Marquardt approach. Inverse Problems in Science and Engineering 29 (12):2207–2238, 2021.
- [2] X. Jin, G. Yin, N. Chen. Advanced estimation techniques for vehicle system dynamic state: a survey. Sensors 19, 4289, 2019.
- [3] R.N. Jazar. Vehicle dynamics: theory and application. 3rd edition, Springer International Publishing AG, 2017.
- [4] K.B. Singh, M.A. Arat, S. Taheri. Literature review and fundamental approaches for vehicle and tire state estimation. Vehicle System Dynamics, doi.org/10.1080/00423114.2018.1544373, 2018.
- [5] C. Angrick, S. van Putten, G. Prokop. Influence of tire core and surface temperature on lateral tire characteristics. SAE International Journal of Passenger Cars – Mechanical Systems, 7:468–481, 2014.
- [6] E. Askari, G. Crevecoeur. Evolutionary sparse data-driven discovery of complex multibody system dynamics. Multibody System Dynamics, (in press), 2022.