Dynamics control of a spatial spacecraft-manipulator system based on geometric approaches

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EXTENDED ABSTRACT

1 Introduction

Spacecraft equipped with a dexterous manipulator is crucial for many future space missions, such as debris removal and on-orbit assembly. In these missions, one basic but necessary task is that the end-effector of the manipulator needs to track a specific planned trajectory, as shown in Figure 1. However, control of such a floating spacecraft-manipulator system presents new challenges compared with well-researched fix-base manipulators. Since the base of space manipulator is not fixed to the ground, the motion of the manipulator's joints leads to reaction motion of the base spacecraft. This dynamics coupling effect is not negligible and even dominant, when the mass and inertia ratios of the base spacecraft to the manipulator are of the same magnitude. To simulate and control the spacecraft-manipulator system, previous studies [1, 2] usually use specific rotation parameterizations, such as Euler angles and unit quaternions, to describe the orientations of the rigid bodies of these multibody systems. However, such parameterizations lead to not only numerical singularity issues, but also inaccurate integration results. To address these problems, a different but general dynamic modelling and control frame for the spacecraft-manipulator system is established in this study. Based on the inherent geometric nature of spatial rigid body motion, the twists from the screw theory, which is analogous to the lie algebra element from the Lie group theory, is used to formulate the dynamic equations and control algorithms.

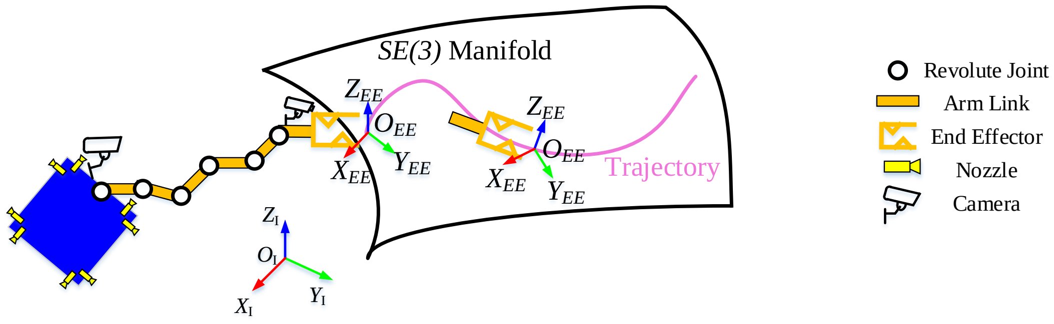

Figure 1: Spatial spacecraft-manipulator system

2 Hybrid twists and dynamic equations

In order to derive a recursive formulae of differential kinematics, the "hybrid" twist [3] of body i in the spacecraft-manipulator system is defined by

$$\boldsymbol{t}_i = \left[\left(\left(\dot{\boldsymbol{R}} \boldsymbol{R}^{\mathrm{T}} \right)^{\vee} \right)^{\mathrm{T}} \quad \left(\dot{\boldsymbol{p}} \right)^{\mathrm{T}} \right]^{\mathrm{T}} = \left[\left(\boldsymbol{\omega}_s \right)^{\mathrm{T}} \quad \left(\boldsymbol{R} \boldsymbol{v}_b \right)^{\mathrm{T}} \right]^{\mathrm{T}}, \tag{1}$$

where $\boldsymbol{R}$ is the rotation matrix of body i with respect to the inertial frame, $\boldsymbol{p}$ is the displacement vector of origin O_i of body i's body-fixed frame, $\boldsymbol{\omega}_s$ is the angular velocities of body i, $\boldsymbol{v}_b$ is the translational velocities of origin O_i, and all these terms are resolved in the inertial frame. Then the recursive formula of each body twist can be obtained by propagating the twists according to the joint pair relations, starting from the base spacecraft (body 0) to the end effector (body n). Furthermore, the dynamic equations of system and Jacobian of the end-effector can be derived in terms of the twists. To design the control algorithms in the next section, the succinct form of the system momentum can be written as

$$\begin{aligned}
&\text{free-floating}: \begin{bmatrix} \boldsymbol{P}(t) \\ \boldsymbol{L}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{0\mathrm{r}} & \boldsymbol{H}_{0\theta\mathrm{r}}^{\mathrm{T}} \\ \boldsymbol{H}_{0\theta\mathrm{r}} & \boldsymbol{H}_{0\theta} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{r}}_0 \\ \boldsymbol{\omega}_0 \end{bmatrix} + \begin{bmatrix} \boldsymbol{H}_{0\mathrm{rm}} \\ \boldsymbol{H}_{0\theta\mathrm{m}} \end{bmatrix} \dot{\boldsymbol{q}}_{\mathrm{m}} = \begin{bmatrix} \boldsymbol{P}_0 \\ \boldsymbol{L}_0 \end{bmatrix} \\
&\text{free-flying}: \begin{bmatrix} \boldsymbol{P}(t) \\ \boldsymbol{L}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{0\mathrm{r}} & \boldsymbol{H}_{0\theta\mathrm{r}}^{\mathrm{T}} \\ \boldsymbol{H}_{0\theta\mathrm{r}} + \boldsymbol{r}_0 \times \boldsymbol{H}_{0\mathrm{r}} & \boldsymbol{H}_{0\theta} + \boldsymbol{r}_0 \times \boldsymbol{H}_{0\theta\mathrm{r}}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{r}}_0 \\ \boldsymbol{\omega}_0 \end{bmatrix} + \begin{bmatrix} \boldsymbol{H}_{0\mathrm{rm}} \\ \boldsymbol{H}_{0\theta\mathrm{m}} + \boldsymbol{r}_0 \times \boldsymbol{H}_{0\mathrm{rm}} \end{bmatrix} \dot{\boldsymbol{q}}_{\mathrm{m}} = \begin{bmatrix} \int \boldsymbol{F}_{\mathrm{thrust}} \mathrm{d}t \\ \int \left(\boldsymbol{T}_{\mathrm{thrust}} + \boldsymbol{r}_0 \times \boldsymbol{F}_{\mathrm{thrust}} \right) \mathrm{d}t \end{bmatrix}
\end{aligned}, \tag{2}$$

where the free-floating states means that the system momentum is conserved, and the free-flying state means that the nozzles and momentum wheels of the base-spacecraft can apply wrenches (thrusts and torques) to the system.

3 Control algorithms

In this section, two control algorithms are considered for controlling the configuration of the end-effector in co-simulation. The first control algorithm is the twist-space PI controller of the end-effector as follows

$$\boldsymbol{t} = \boldsymbol{t}_{\mathrm{d}} + K_{\mathrm{p}}\boldsymbol{t}_{\mathrm{e}} + K_{\mathrm{i}}\int \boldsymbol{t}_{\mathrm{e}}\mathrm{d}t, \quad \boldsymbol{t}_{\mathrm{e}} = \boldsymbol{T}_{\mathrm{d}} \ominus \boldsymbol{T} = \mathrm{Log}(\boldsymbol{T}_{\mathrm{d}} \circ \boldsymbol{T}^{-1}) \tag{3}$$

where $\boldsymbol{t}_{\mathrm{d}}$ is the feedforward twist determined from the inverse kinematics, $\boldsymbol{t}_{e}$ is the error twist, $\boldsymbol{T}_{\mathrm{d}}$ and $\boldsymbol{T}$ are the desired and real configurations of the end-effector, respectively, and Log is the logarithmic mapping of SE(3) group. Different from classic PI controller on Euclidian space, Eq. (3) gives a feedback controller on Lie group, where the error twist is defined as the six-axis pose difference projected on the tangent space of manifold. This Lie group controller has better convergence rates and only one group of control parameters to be tuned. The second control algorithm is the nonlinear model predictive controller (NMPC), which solves the following optimal control problem:

$$\begin{aligned} &\Psi(\mathbf{x},\mathbf{u}) = \sum_{k=0}^{N-1} \left\| \mathbf{x}_k \ominus \mathbf{x}_k^{\mathrm{r}} \right\|_{\mathbf{Q}}^2 + \left\| \mathbf{u}_k - \mathbf{u}_k^{\mathrm{r}} \right\|_{\mathbf{R}}^2, \\ &\text{s.t. } \mathbf{x}_0 = \mathbf{x}_{\mathrm{init}}, \mathbf{x}_{k+1} = \mathbf{f}\left(\mathbf{x}_k, \mathbf{u}_k\right), \mathbf{u}_k \in U, \mathbf{x}_k \in X, \quad k = 0,1,\ldots N \end{aligned} \tag{4}$$

where $\Psi(\mathbf{x},\mathbf{u})$ is the cost function, $\mathbf{Q}$ and $\mathbf{R}$ are weight matrices of the state variables $\mathbf{x}$ and control inputs $\mathbf{u}$, respectively, and the constraint equations are related to the initial conditions, time integrations, control constraints and states constraints.

4 Results

Figure 2 shows the trajectory tracking results of co-simulation and the experimental setup in our laboratory. The proposed control algorithms have shown to be effective and accurate for the trajectory tracking problem. If the system is at free-floating state, the proposed twist-space PI controller is simple to implement and enables the smooth tracking of a slow trajectory. When a fast and accurate tracking of certain trajectory is required, the NMPC shows better performance by introducing nozzle thrusts as additional control inputs. Also, the multiple shooting method is used to convert the optimal control problem in Section 3 to a nonlinear programming problem. The results show that when a reasonable large prediction horizon is chosen, the computational efficiency can meet the real-time challenge posed by online controller hardware. Further researches will aim on the momentum-energy conservation characteristics of the system in a controller framework, by using the the discrete mechanics optimal control and other geometric approaches.

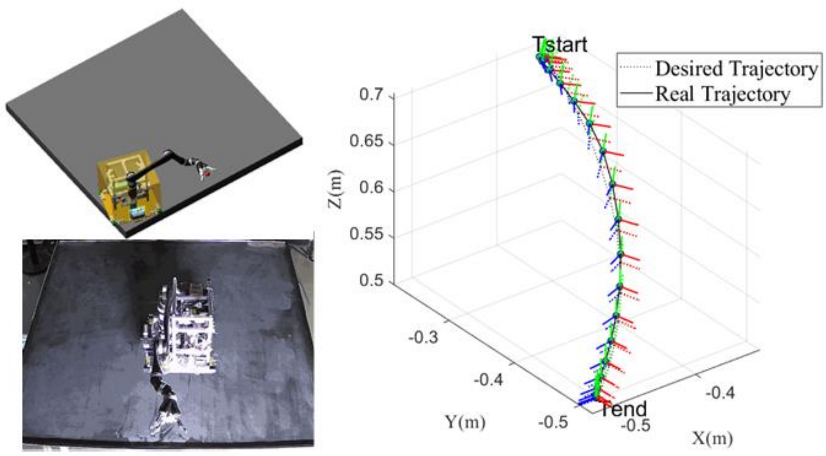

Figure 2: Results of co-simulation and the ground experimental setup in laboratory

Acknowledgments

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References

[1] M. Wilde, S.K. Choon, et al. Equations of motion of free-floating spacecraft-manipulator systems: an engineer's tutorial. Frontiers in Robotics and AI, 5:41, 2018.

[2] K. Seweryn, F.L. Basmadji, T. Rybus. Space robot performance during tangent capture of an uncontrolled target satellite. The Journal of the Astronautical Sciences, 69(4):1017-1047, 2022.

[3] A. Müller. Screw and Lie group theory in multibody kinematics: Motion representation and recursive kinematics of tree-topology systems. Multibody System Dynamics, 43(1): 37-70, 2018.