Advanced Pseudo-Rigid Body Models for the Design of Compliant Mechanisms

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EXTENDED ABSTRACT

1 Introduction

Compliant mechanisms are designed to transmit motion and forces as rigid-body mechanisms. They are used in many technical fields, from actuator mechanics and robotics to micro-mechanisms and also as a simple replacement for standard rigid-link mechanisms [1]. A compliant mechanism generally has an elasto-kinematic characteristic, *i.e.*, the kinematic behaviour is influenced by the acting forces, unlike the traditional rigid-body mechanisms for which the kinematics is independent of the applied loads. This peculiarity complicates the design as kinematics, statics and dynamics must be studied with a multidisciplinary approach. Compliant mechanisms with flexible hinges are often studied through the so-called pseudo-rigid body (PRB) arrangements [2]. They are mechanisms composed of rigid bodies and concentrated springs able to reproduce the elasto-kinematic behaviour without the burden of taking into account the detailed deformation characteristics. PRB models can then be studied using multibody techniques to address synthesis, analysis and optimization problems with a reduced computational effort with respect to finite element or flexible multibody approaches [3]. They represent a direct link between compliant mechanisms and multibody dynamics.

Most of the PRB models approximating the behaviour of elastic hinges are based on the first idea of embodiment, whose dimensions are then optimized to reproduce the elasto-kinematic behaviour in the range of motion and boundary conditions of interest. The purpose of the lecture is to give an extended overview of the building of advanced pseudo-rigid body models of flexure hinges starting from the study and interpretation of the kinematic invariants of motion. The embodiment is then a consequence of the observed elasto-kinematic behaviour rather than guessed at the beginning and then optimized. This allows for simpler models with reduced degrees of freedom and excellent accuracy.

The lecture first recalls the basics of kinematic invariants and discusses the elasto-kinematic intrinsic properties of the most common flexure hinges. For most of them, parametric embodiments ready to be integrated into complex multibody models are deduced and discussed. Complete examples of kinematic synthesis and dynamic analysis will also be presented in order to show the potential of the approach in different engineering fields including the building of digital twins.

2 Kinematic invariants to study the relative motion between connected parts

Flexure hinges are elements that connect two rigid parts, preserving the ability of a relative motion between them due to their compliance. Investigating their elasto-kinematic features can be challenged by studying and interpreting the relative motion between the corresponding parts on the basis, for example, of instantaneous kinematic invariants, namely, the fixed and moving centrodes and the diameter of the inflection circle [4]. The centrodes are the paths traced by the instantaneous center of rotation of a rigid body moving in a plane. The fixed centrode refers to the trace in the fixed reference frame, the moving one refers to a reference system attached to the moving body. For a rigid body moving in a plane, the inflection circle is the locus of the points whose acceleration is parallel to velocity (their normal acceleration vanishes). Therefore, the trajectory of the inflection circle points has zero curvature. The rolling without slipping of the moving centrode with respect to the fixed centrode reproduces the relative motion between two bodies. Using the kinematic invariants, the relative motion between the connected bodies may be studied independently from the actual topological arrangement (rigid or flexible) that supports it. Moreover, since the inflection circle incorporates information about the relative acceleration, it is a second-order invariant and can be considered a very reliable descriptor of the relative motion properties.

A preliminary study on five common types of flexure hinges (Figure 1) showed that, in the case of a pure moment applied to the moving body, the circular profile, the elliptical profile and the leaf flexure hinge are capable of reproducing a relative motion with an inflection circle with an almost constant radius [5].

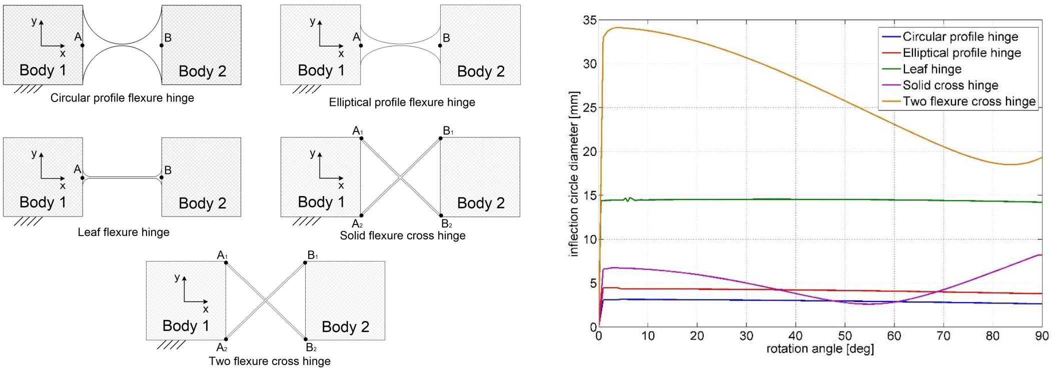
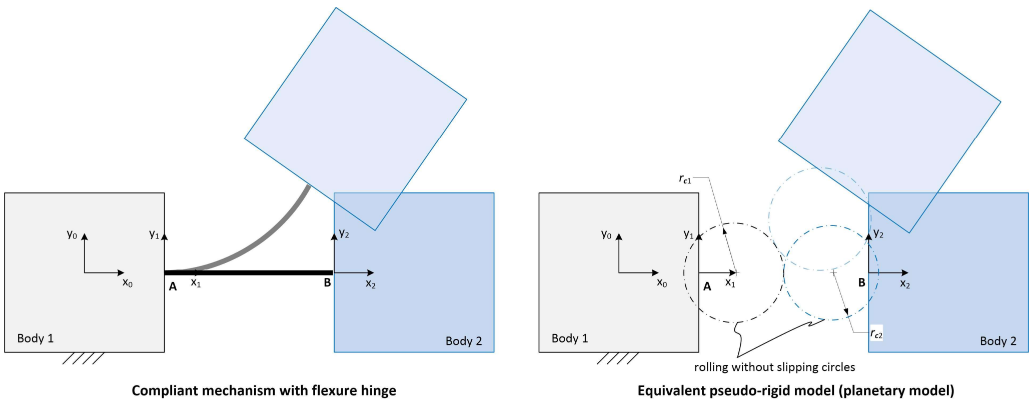


Figure 1. Preliminary investigation on the elasto-kinematic invariants of five common flexure hinges subjected to pure moment.

3 Pseudo-rigid embodiments

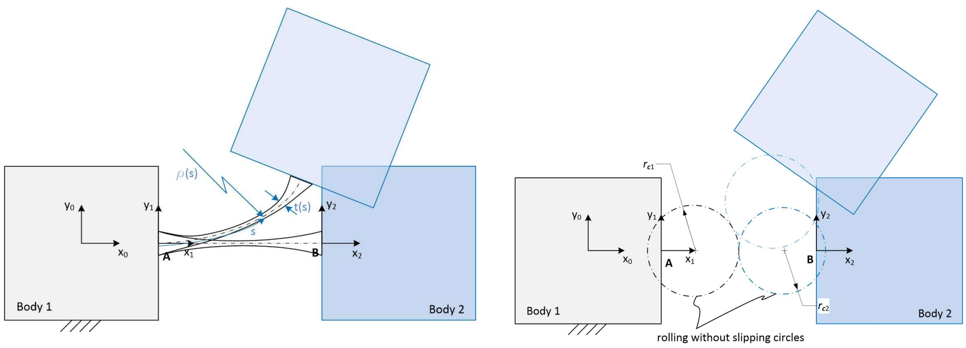
Based on the results of the preliminary study, the relative motion of the two bodies connected by the three types of hinges with an almost constant radius of the inflection circle can be replicated as rolling without slipping of two circles with radius proportional to geometrical features of the flexible element (Figure 2). An analytical approach allowed us to deduce a closed formula for the dimensioning of the planetary arrangement for the leaf flexure hinge that depends only on the length of the flexible part l [6]. The proposed pseudo-rigid model is able to approximate the elasto-kinematics of the leaf flexure hinge in a wide range of motion, being suitable for large displacement investigations.



$$r_{c_1} = r_{c_2} = 2\delta \approx \frac{\pi}{10} l$$

Figure 2. The proposed pseudo-rigid mechanisms of the leaf flexure hinge.

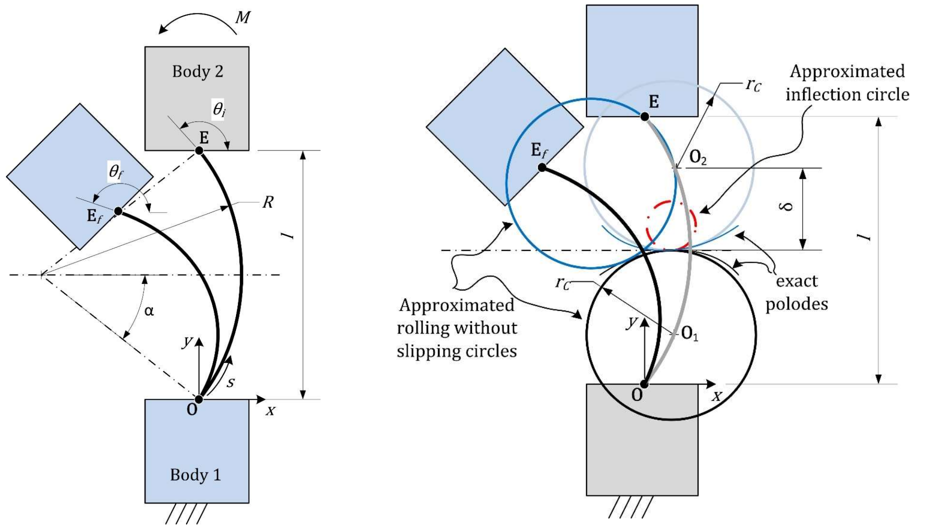
The results of the leaf flexure hinge have also been extended to the cases of the parabolic flexure hinge (Figure 3) and the circular arc flexure hinge whose relative motion can be approximated by the same embodiments but with different dimensions [7].



$$r_{c1} = r_{c2} = r_c = \frac{\pi}{10} l \sqrt{\frac{t_{\min}}{t_{\max}}}$$

Figure 3. The proposed pseudo-rigid mechanisms of the parabolic flexure hinge.

In the case of parabolic flexure hinge, the relative motion can be approximated by the same embodiments but with different dimensions depending also on the ratio between the minimum and maximum thickness of the cross section. In the case of a circular arc, an offset of the center of the rolling circles must be introduced [8] (Figure 4).



$$\mathbf{O}_1 = \begin{Bmatrix} \dfrac{l}{2}\left(\dfrac{1}{\alpha}-\dfrac{1}{\tan\alpha}\right) \\ \dfrac{l}{2}-\left(\dfrac{3\pi l}{10}\left(\dfrac{1}{\alpha^2}-\dfrac{1}{\alpha\tan\alpha}\right)\right) \end{Bmatrix}$$

$$\mathbf{O}_2 = \begin{Bmatrix} \dfrac{l}{2}\left(\dfrac{1}{\alpha}-\dfrac{1}{\tan\alpha}\right) \\ \dfrac{l}{2}+\left(\dfrac{3\pi l}{10}\left(\dfrac{1}{\alpha^2}-\dfrac{1}{\alpha\tan\alpha}\right)\right) \end{Bmatrix}$$

$$r_C = 2\overline{\delta} = \frac{3\pi l}{10}\left(\frac{1}{\alpha^2}-\frac{1}{\alpha\tan\alpha}\right)$$

Figure 4. The proposed pseudo-rigid mechanisms of the circular arc flexure hinge.

4 Verification and examples of application

The proposed embodiments have been verified against finite element models and flexible multibody models, and they always presented a very good accuracy in an extensive range of motion. Furthermore, the proposed models have been tested in some practical cases of the synthesis of a compliant four-bar mechanism [9] (Figure 5) and a compliant centrifugal vibration damper [10].

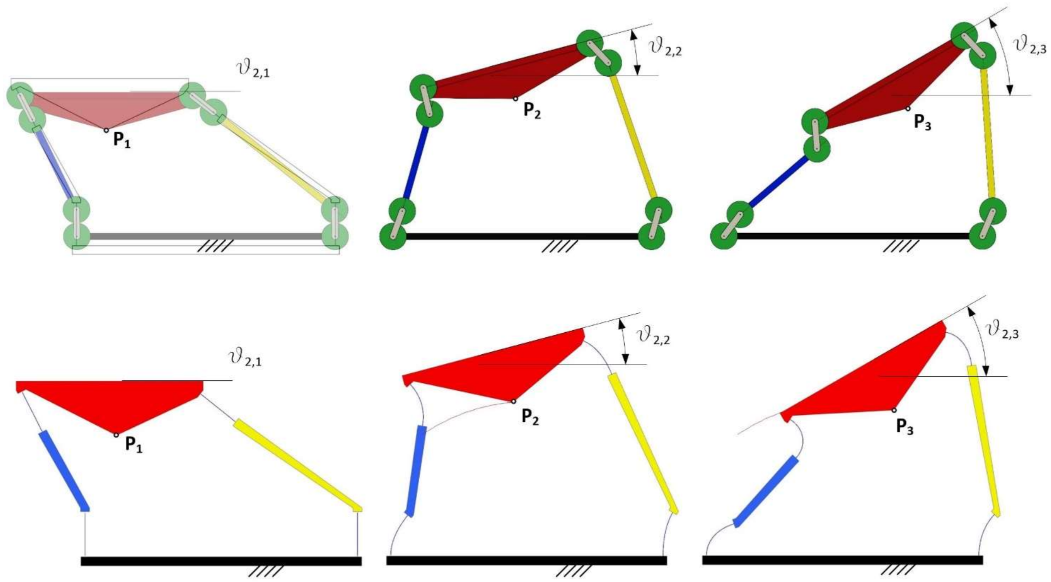


Figure 5: A fully compliant four-bar linkage dimensioned using the planetary rigid body arrangement, preserving a single degree of freedom.

5 Conclusion

Starting from the knowledge of the relative motion characteristics described in terms of kinematics invariants it is possible to develop accurate and compact pseudo-rigid body mechanisms to be used in the synthesis and design of compliant mechanisms. Thanks to the rigid body schematization, the models can be implemented using standard rigid multibody models without the need for complex flexible systems but preserving the accuracy in motion simulation even for large displacement. The model may be used for facing problems of both synthesis and analysis. For many leaf-like flexure hinges, the pseudo-rigid body model includes two circles with the same radius that roll on each other without slipping and one torsion spring. The equivalent mechanism can be dimensioned starting from the study of the inflection circle diameter, which remains almost constant for a very large range of motion. The proposed models can be also successfully used for interactive real-time visualization of digital twins (in virtual and augmented reality scenarios) in order to implement rigid body rigging that guides more complex flexible mesh (skinning) preserving realism and accuracy.

Although the proposed pseudo-rigid models are deduced under the hypothesis of a pure applied moment, additional simulations showed that the epicyclic arrangement could be potentially extended to different load combinations.

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