Efficient Computational Method for Multibody Dynamics of Supersonic Intermittent Contact System

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ABSTRACT

To balance computational efficiency and accuracy when simulating the multibody dynamics of a supersonic intermittent contact system, a modeling and computational method was proposed based on the finite volume beam elements and deformable slider joints. The errors in natural frequencies between the simulation model and modal test were within an acceptable range. The simulation results demonstrated that the model, containing over 6,000 differential-algebraic equations, can be computed in approximately 0.076 seconds per time step, which was considered acceptable. This study confirmed that track vibration has a significant impact on the dynamic response of the slipper, even though the magnitude of the position of the track is small. The results also indicate that track vibration can be transmitted to other positions of the track, highlighting the importance of treating the track as a flexible body. This study improved computational efficiency of the intermittently contacted long-distance flexible track, which provided a foundation for future research on the multibody dynamics of the supersonic rocket sled and the flexible track under intermittent contacts.

Keywords: Multibody dynamics modeling, Efficient computational method, Finite volume beam, Deformable slider joint, Supersonic intermittent contact system.

1 INTRODUCTION

A supersonic rocket sled is a large and high-precision ground test equipment. Supersonic rocket sled tests use a rocket engine to push the sled along a track at high speed. Meanwhile, the performance of the test specimen mounted on the sled can be tested [1-2]. The component of the rocket sled that contacts the track is called *slipper*. The slipper wraps around the track, keeping the sled from flying off the track as it moves, which is shown in Fig. 1. The slipper and the track contact intermittently with a series of impacts. Therefore, the slipper-track system is a supersonic intermittent contact system. Lots of studies showed that supersonic intermittent contact can cause severe vibration of the rocket sled and significantly affects its motion stability. Therefore, its study is very important.

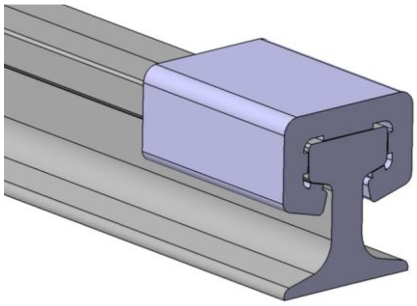

Figure 1. *Slipper and track.*

Some studies have already been conducted on the multibody dynamics of supersonic intermittent contact system. Usually, the track is regarded as a rigid body, and the position constraints of the slipper are established according to the track irregularity; the dynamic response of the rocket sled is studied on this basis [3]. The track is often regarded as a rigid body due to computational considerations, because it needs to be very long, owing to the high speed of the sled. However, this method ignores the influence of track vibration on the slipper. The slipper and the track are in intermittent contact and interact with each other, so it is more reasonable to regard the track as a flexible body. There are two ways to establish the flexible dynamic model of the rocket sled track in other studies. The first is to discretize the track using the finite element method (FEM) [4]. The second is to regard the track as a continuous beam model and use the modal analysis method based on the normal modes of the track [5]. However, these two methods require a long time for computation.

To sum up, for the multibody dynamics simulation of supersonic intermittent contact system, the difficulty is how to balance the computational efficiency and accuracy of track model. Since the track is a typical beam structure, using the finite volume method (FVM) to discretize the track becomes an idea to solve the problem. Reference [6,7] proved the high efficiency of using FVM to discretize beam in multibody dynamics computations. In this method, internal forces of a beam are evaluated only at the boundaries of the volumes, thus simplifying their contribution to the equilibrium equations. In addition, Morandini [8] established a method to make the constrained point move along a series of finite volume beams. Therefore, this study will propose a modeling and computational method for supersonic intermittent contact system based on finite volume beams and slider joints. This approach will improve the computational efficiency of the intermittently contacted long-distance flexible track.

2 METHOD

2.1 Modeling of track

The track model utilized the finite volume beam element, which has been incorporated into the MBDyn framework to simulate the elastic deformation of beams subjected to large displacements and rotations. MBDyn is a multibody analysis program developed at the Department of Aerospace Engineering of the Politecnico di Milano [9]. Deformable beams can be interpreted as discrete elastic constraints that link independent rigid bodies. The three-node beam element is used to model the track. A piece of beam is divided in three parts that are related to three reference points, which includes the midpoint and the two endpoints. The beam element is divided in finite influence regions surrounding the nodes. The boundaries between the influence regions are the so-called *evaluation points*. At each evaluation point, a 6D constitutive law is defined. It defines the relationship between the generalized beam strains and their time derivatives and the internal forces and moments at the evaluation points.

To compare with the experimental natural frequencies of the track, 10 modules of track are first established. The track model includes tracks, blocks, joints, and the ground. Among them, there are 10 track modules and 11 blocks. Each module of track beam is connected by a common node. Different meshes with one, two and four beams for each track module are considered. The eigen analysis was performed using MBDyn, which is capable of directly addressing the eigen analysis of the system of equations resulting from linearizing the equations that describe the constrained dynamics of generic systems [10]. Table 1 presented the first four natural frequencies of three types of models, as well as the modal test results. Comparison of the results shows that both the two-beam and the four-beam model have acceptable errors. Therefore, the two-beam model is chosen as the best compromise between accuracy and the computational requirements.

Table 1. *The first four natural frequencies of different models and modal test*

Model \ Order	1st / Hz	2nd / Hz	3rd / Hz	4th / Hz
one-beam	476.3	483.2	494.9	511.7
two-beam	425.2	430.8	440.2	453.4
four-beam	423.0	428.6	438.1	451.5
modal test	423.9	427.7	432.3	436.1

2.2 Modeling of supersonic intermittent contact system

The study of the supersonic intermittent contact system has been implemented in the MBDyn. To this end, a total of 100 track modules are considered for the supersonic intermittent contact system, as illustrated in Fig. 2. The length of the track is 100 meters. The slipper in the supersonic intermittent contact system is considered as a rigid body, since its stiffness is much larger than that of track. The initial velocities of the slipper in three directions are equal to 500 m/s, 1 m/s, and 1 m/s, respectively, while the initial angular velocities were equal to 0.5π rad/s, 0.4π rad/s, and 0.8π rad/s, respectively. Gravity acted in the -*z* direction. The beam-slider joint is used to connect the track and two sliders. The slider node is static without mass and inertia. There are 20 contact points between the slipper and the track. The initial gaps in the *y* and *z* directions between the slipper and the track were both set to 0.002 m. Therefore, using the offsets of the slipper and two sliders, the normal contact forces can be defined considering the gap between them.

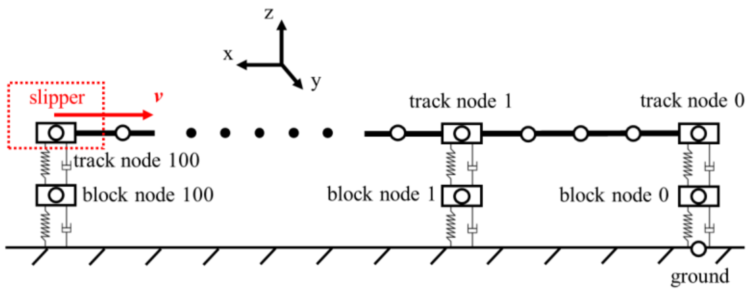

Figure 2. *Model of supersonic intermittent contact system.*

The normal contact force is defined by the contact stiffness and damping, with respect to the relative position and velocity [11]. This model is considered dissipative, as it considers energy loss during contact, expressed as

$$F = Kg^{e} + step(g, DMAX)c\dot{g}\ , \tag{1}$$

where *K* is the generalized contact stiffness, *g* is the relative position, $\dot{g}$ is the relative velocity, *e* is the exponent, *c* is the damping constant, *DMAX* is the maximum penetration depth at which the damping force is scaled to $c\dot{g}$ with a cubic step function, and *step*(*g,DMAX*) is a cubic step function that smoothly increases the damping constant from zero to *c* as the penetration increases from zero to *DMAX*.

In addition, the frictional forces in three directions are considered as

$$f = step(v, v_{thres})F\mu\ , \tag{2}$$

where μ is the kinetic friction coefficient, v_{thres} is the threshold velocity, and $step(v, v_{thres})$ is a cubic step function that smoothly increases from zero to one as the relative tangential velocity increases from zero to v_{thres}.

The overall simulation time is 0.2 s, with a time step of 10^{-4} s. The integration algorithm is the backward differentiation formula (BDF) [12], which is an implicit multi-step integration algorithm with variable order and variable step size.

3 RESULTS AND DISCUSSION

The simulation of a flexible track of 100 m, which consisted of 502 nodes, 200 beam elements, and 202 viscoelastic supports, required a total of 152.19 seconds of CPU time. The resulting model contained over 6,000 differential-algebraic equations, and the computation time required per time step was approximately 0.076 seconds. This ratio of computation time to simulated time resulted in a wall clock and simulated time ratio of approximately 760, which was considered acceptable.

Throughout this paper, the x direction refers to the sliding direction, which is indicated by blue lines in the following figures. The y direction represents the lateral direction, indicated by yellow lines, while the z direction denotes the vertical direction, indicated by red lines. Fig. 3 illustrated the dynamic response of the slipper, including its accelerations, velocities, positions, and Euler angles in three dimensions. Over the course of the simulation, the slipper decelerated from 500 m/s to 495.7 m/s due to frictional forces and traveled a total distance of 99.5 m after 0.2 seconds. Because the normal contact force model included a damping term, the vertical and lateral velocities of the slipper tended to decrease after intermittent contacts. However, the velocity in the z direction briefly increased at t = 0.12 s due to vibration of the track. Additionally, the decreasing normal contact force and frictional force caused the negative acceleration in the x direction to decrease. The acceleration, velocity, and position in the z direction were larger than those in the y direction because the vertical and pitch stiffness of the deformable joint between the track and the ground was greater than the lateral and yaw stiffness. The Euler angles of the slipper remained below 1 degree due to the small gap between the slipper and track. Frequent positive and negative changes in Euler angles indicated that different parts of the slipper were in contact with the track, with the Euler angle in the y direction changing direction most frequently. This suggested that the front and rear of the slipper collided with the track most frequently in the z direction.

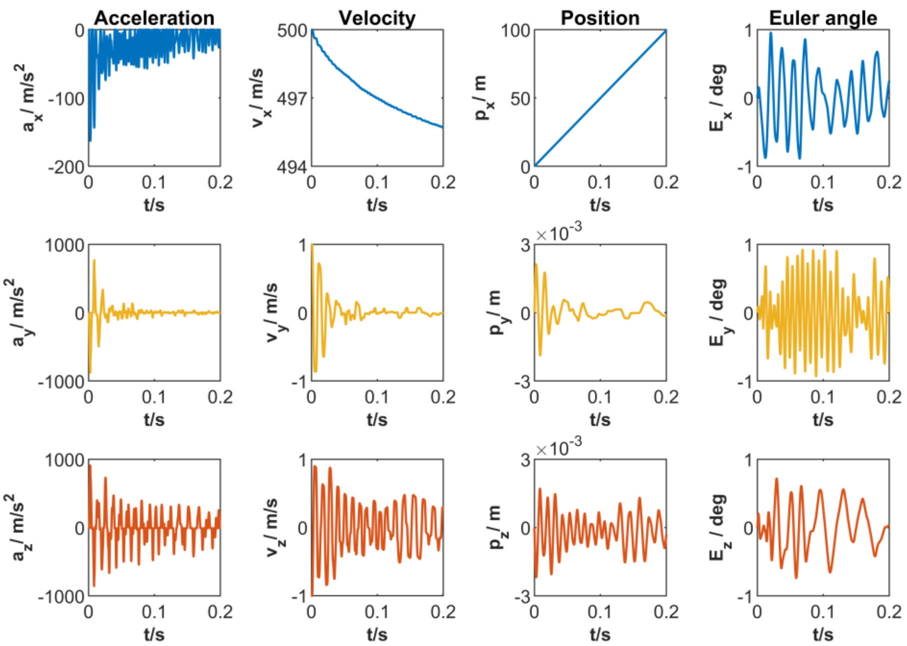

Figure 3. *Dynamic response of slipper.*

The positions of slider nodes correspond to the positions of the track where the slipper is located in both the vertical and lateral directions, as slider nodes are always located along the track beam. Fig. 4 illustrated the vertical and lateral positions of the front slider node. Initially, the track beam experienced greater vibration in the y direction, but after t = 0.03 s, the position of the track beam was larger in the z direction. This corresponds to the vertical and lateral positions of the slipper shown in Fig. 3, indicating that track vibration has a significant impact on the dynamic response of the slipper, even though the magnitude of the position of the track is small.

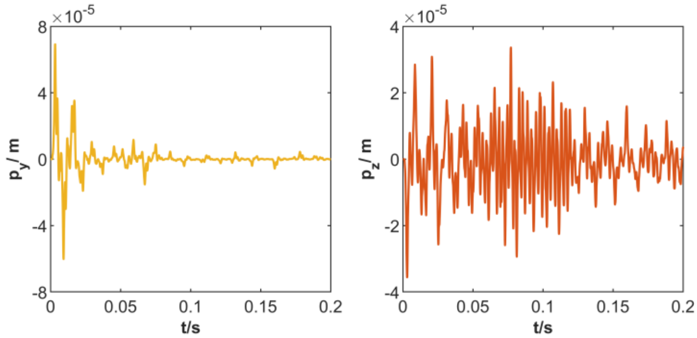

Figure 4. *Positions of front slider node.*

Figure 5. *Vertical positions of each middle node of track module.*

Fig. 5 illustrated the vertical positions of each middle node of the track module. The figure demonstrated that as the slipper passed over the track, the track at that point experienced significant vibration. Moreover, this vibration can be transmitted to other positions of the track, highlighting the importance of treating the track as a flexible body. Furthermore, the maximum value of the vertical positions of these 100 track modules corresponded to the vertical position of the slider node depicted in Fig. 4.

4 CONCLUSIONS

This study discussed the modeling and simulation of a supersonic intermittent contact system based on finite volume beams and deformable slider joints. This approach improved the computational efficiency of the intermittently contacted long-distance flexible track while maintaining accuracy.

The natural frequency errors between the simulation model and modal test were found to be within an acceptable range. The simulation results showed that the model, which involved more than 6,000 differential-algebraic equations, could be computed in approximately 0.076 seconds per time step, which was deemed acceptable. Importantly, the study found that track vibration has a significant impact on the dynamic response of the slipper, despite the small magnitude of the position of the track. The results also revealed that track vibration can be transmitted to other positions of the track, emphasizing the importance of treating the track as a flexible body.

Overall, this study laid the groundwork for future research on the multibody dynamics of the supersonic rocket sled and the flexible track under intermittent contacts.

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