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Friction Modeling from a Practical Point of View

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ABSTRACT

In this paper, the standard friction models of the commercial multibody simulation packages Adams, RecurDyn and Simpack are clarified and compared with a practical example. In addition to the standard regularization of the friction characteristics, the focus is on specific stick-slip models as well. Two different approaches are used by the packages to represent long-term sticking and sliding. The paper gives an overview of the functionality of these friction models and shows their behavior in detail.

Keywords: Multibody Dynamics, Stick-Slip-Effect, Adams, RecurDyn, Simpack.

1 INTRODUCTION

A large number of different friction models are presented in the current literature. Today, commercial multibody simulation (mbs) packages such as Adams, RecurDyn or Simpack offer a limited choice of specific friction models, in particular specific stick-slip approaches for joint friction [1, 2, 3]. For instance, Adams users can choose between a regularized static friction model, the dynamic LuGre model and an Adams specific friction model to describe stick-slip [1]. This paper applies a practical example to test these friction models regarding to their reproducibility of friction phenomena and user-friendliness.

2 GENERAL FRICTION CHARACTERISTICS

In general, the frictional force has a static and a dynamic part. According to Coulomb, a critical static friction force must be exceeded to set a frictional body in motion. If this body is in motion, a dynamic friction force acts [4, p. 156f]. Here, the friction force F_R is proportional to the normal force via the friction coefficient μ, see Fig. 1. According to Stribeck, hydrodynamic friction also shows a velocity dependent friction force.

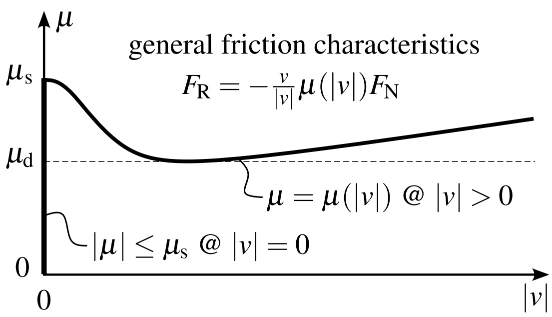


Figure 1. General friction characteristics with discontinuity at $|v| = 0$

Therefore, friction characteristics are usually considered as a function of velocity. Fig. 1 shows this behavior, where μ_s describes the coefficient of static friction and μ_d the coefficient of dynamic friction. For stiction ($v = 0$), this function is ambiguous, since the actual friction force acting in this case depends on the external force. To set the body in motion, the static friction level must be exceeded. For $|v| > 0$, a velocity dependent friction coefficient generally applies.

3 FRICTION MODELS IN COMMERCIAL MULTIBODY SIMULATION PACKAGES

The simplest model provided by these mbs packages is a piecewise defined regularization between friction regimes. A standard regularization approximates stiction behavior by slow joint creep. To achieve long-term stiction, mbs package specific friction models are provided. These models switch between stick and slip. Each algorithm uses relative velocity to distinguish between different states to maintain long-term stick [1, 2, 3]. Adams also offers the LuGre model as a dynamic friction model. However, as illustrated in [5], this model exhibits severe drawbacks.

3.1 Standard Regularizations of Adams and RecurDyn

All three mbs packages offer a regularization of the friction characteristic shown in Fig. 2. A regularized friction model $\mu = \mu(|v|)$ is usually defined by three characteristic points: $(\mu(|v| = 0) = 0)$, $(\mu(|v| = v_s) = \mu_s)$, and $(\mu(|v| = v_d) = \mu_d)$, where μ_s and μ_d specify the static and dynamic friction coefficient and the velocities v_s and v_d model the regularization and the attenuation pattern.

In friction regularization, the friction coefficient μ is calculated by a piecewise defined function, depending on the relative velocity v. In the first section for $|v| \leq v_s$, the friction coefficient $\mu(|v|)$ increases from $\mu(0) = 0$ to the stiction coefficient $\mu(v_s) = \mu_s$. In the second section for $v_s < |v| \leq v_d$, $\mu(|v|)$ decreases from $\mu(v_s) = \mu_s$ to the dynamic coefficient $\mu(v_d) = \mu_d$ and in the third section for $v_d < |v|$, the friction coefficient applies to $\mu(|v|) = \mu_d = \text{const}$.

To approximate the friction characteristics, see Fig. 1, the best possible way, $v_s \to 0$ and therefore has a very small value. To generate a frictional force the body needs a relative velocity.

In Adams and in RecurDyn a 5th order polynomial (STEP5 function) is used to describe the smooth transition between the friction coefficients [1, 3]. In contrast, Simpack models its regularization with trigonometric functions [2]. Figure 2 shows a general plot comparing the regularizations.

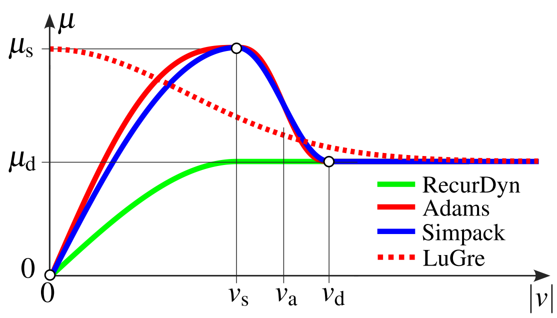


Figure 2. Regularized friction characteristics $\mu = \mu(|v|)$ and the LuGre approach

The STEP5 function is described in its typically syntax as used in the mbs packages. For example, STEP5(x, x_0, y_0, x_1, y_1) is a 5th order polynomial that smoothly changes the value y_0 to y_1 in the interval $x_0 \leq x \leq x_1$.

3.2 LuGre Model

The LuGre model is a dynamic friction model that is often described in the literature. It bases on a bristle model which describes dynamic friction force [6]. The Adams implementation also takes a normal force dependency into account [1]. The advantages and disadvantages of this description are well known and can be found in Åström et al. [7], Marques et al. [8] and Rill et al. [5], among others.

The main disadvantages are the drift during pulse-like excitation and the undefined frictional characteristics caused by the bristle dynamics and the discontinuous steady-state friction characteristics.

The LuGre model uses the fictitious velocity $v_a > 0$ and the exponent $\alpha = 2$ as a standard to describe the transition from the static friction value μ_s to the dynamic friction value μ_d. The steady-state characteristics is typically calculated by the equation

$$\mu_{LG}(|v|) = \mu_d + (\mu_s - \mu_d)e^{-(|v|/v_A)^{\alpha}} \tag{1}$$

and is shown in Fig. 2.

3.3 Standard Stick-Slip Models of Adams and RecurDyn

The specific stick-slip model of Adams and RecurDyn can be separated into $\mu_{stick}(x,v)$ and $\mu_{slip}(v)$. μ_{slip} regularize the transition from μ_s to μ_d by a 5th order polynomial (STEP5 function) and depends only on the relative velocity v [1, 3]. In addition to the standard regularization, the friction coefficient is calculated by a multidimensional function, that also takes into account the relative displacement x. The right two plots in Fig. 3a show the limits of this approach. Where x_s is the regularization displacement, which describes a maximum displacement until the coefficient of friction increases to the static coefficient of friction μ_s. This transition is also modeled by a STEP5 function. The left plot in Fig. 3a show the function of the static friction coefficient $\mu_{stick}(x,v)$. The mathematical model in Adams and RecurDyn follows the equation

$$\mu(x,v) = \begin{cases} \mu_{stick}(x,v), & |v| \leq v_s \\ \mu_{slip}(v) = \pm \begin{cases} \frac{v}{|v|} \cdot \text{STEP5}(|v|, v_s, \mu_s, v_d, \mu_d), & v_s < |v| \leq v_d \\ \frac{v}{|v|} \cdot \mu_d & v_d < |v| \end{cases}, & v_s < |v| \end{cases} \tag{2}$$

Subsequently, the friction force is defined by

$$F_R = \pm \mu(x,v) \cdot F_N \tag{3}$$

In Adams [1] the stiction characteristics is implemented as

$$\mu_{stick,ad}(x,v) = \underbrace{(1 - \beta(|v|))\mu_1(|x|)}_{\mu_{stick,x}} \cdot \text{sign}(|x|) + \underbrace{\mu_s \beta(|v|)}_{\mu_{stick,v}} \cdot \text{sign}(|v|) \tag{4}$$

and in difference to this the stiction characteristics in RecurDyn [3] is implemented by

$$\mu_{stick,rd}(x,v) = -\underbrace{(1 - \beta(|v|))\mu_1(x)}_{\mu_{stick,x}} - \underbrace{\mu_v(v)}_{\mu_{stick,v}} \tag{5}$$

where the last term $\mu_v(v) = \mu_s \beta(v)$. The value $\beta(|v|) = \text{STEP5}(|v|, -v_s, -1, v_s, 1)$ and $\mu_1(|x|) = \text{STEP5}(|x|, -x_s, -\mu_s, x_s, \mu_s)$ are non-linear STEP5 transfer functions, Fig. 3b. The terms $\mu_{stick,x}$ and $\mu_{stick,v}$ are identical in both implementations. The difference is the sign definition, which means that the $\pm$ character in Eq. (2) and Eq. (3) must be adapted to the respective mbs package.

The regularization of stiction over displacement and velocity is shown in Fig. 3a. The upper right plot of Fig. 3a shows the standard regularization for $x = 0$. In the case of $v = 0$, the friction value is determined by the displacement x, as shown in the lower right plot in Fig. 3a.

The first term $\mu_{stick,x}$ is significantly influenced by the relative displacement x, and the second term $\mu_{stick,v}$ is influenced by the relative velocity v. Due to $\mu_{stick,x}$, a coefficient of friction μ can be maintained even without of relative velocity. The term $(1 - \beta(|v|))$ ensures that $\mu_{stick} \leq \mu_s$. In case of “slip to stick” the relative displacement will reset to $x = 0$.

In contrast to the standard regularization, this description makes it possible to create long-term stiction without slipping of the contact bodies. The necessary frictional force is achieved by a small deflection of the bodies.

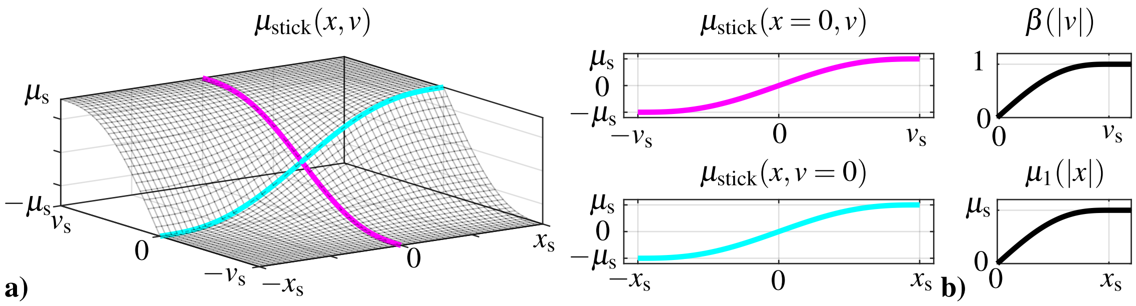


Figure 3. **a)** Stiction region of the stick-slip model of Adams and RecurDyn **b)** Parameters β and μ_1

To parameterize this model, 4 or 5 parameters are required in Adams and RecurDyn, respectively. The friction coefficients μ_s and μ_d, the static regularization displacement x_s, the static regularization velocity v_s and in Adams a transition coefficient λ to describe the dynamic regularization velocity $v_d = \lambda v_s$. By default, $\lambda = 1.5$ in Adams and in RecurDyn $\lambda = 1.5$ is a fixed value that cannot be changed by the user.

Stick-Slip Model of Simpack

The stick-slip model of Simpack is shown in Figure 4. The static friction force is modeled by a spring-damper element with a stiffness c_f and a damping d_f, and the dynamic friction force is the dynamic Coulomb friction force. Simpack distinguishes between stick and slip in the stick to slip direction by the condition $F_{c_f} > \mu_s F_N$ and in the slip to stick direction by the condition $v = 0$. The transition switches directly between the two descriptions of frictional force. In case of slip to stick the spring displacement Δx_c can be reset to zero or preloaded with the sliding force.

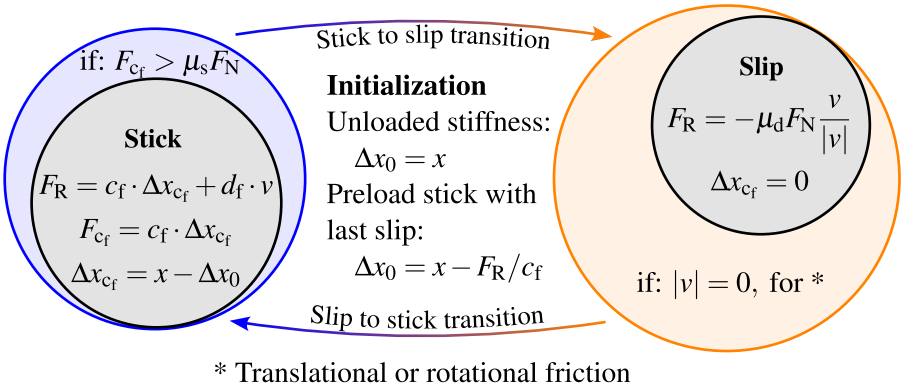


Figure 4. Simpack stick-slip model (refers to [2])

The friction coefficients μ_s and μ_d, the stiffness c_f and the damping d_f are to be defined by the user. The latter two are part of the mathematical model corresponding to a classic penalty approach. The dimensions of c_f and d_f are that's why not comparable to real physical values of stiffness and damping. For non-expert users it is difficult to define the parameters c_f and d_f correctly. One approach to estimate the stiffness

$$c_f = \frac{\bar{F}_s}{x_s} = \frac{\mu_s \bar{F}_N}{x_s} \tag{6}$$

is to use the static friction force $\bar{F}_s$ and a fictitious maximum displacement x_s. In addition, the friction damping parameter d_f can be calculated by

$$d_f = 2D\sqrt{c_f \cdot m_f} \qquad \text{with} \qquad m_f = \frac{\bar{F}_N}{g} \tag{7}$$

using the damping ratio D and m_f is a approximated mass calculated by the estimated normal force $\bar{F}_n$ and the gravity g.

It should be noted that the normal force in multibody systems generally does not have a constant value. For example, $\bar{F}_N$ needs to be estimated by a static equilibrium or from a dynamic simulation.

The stiffness c_f can also be calculated using the rise time t_{rt} needed for the stiffness force F_{c_f} to rise to a constant impulse load. According to [9, p. 316], the rise time

$$t_{rt} = \frac{\pi - \arccos(D)}{\omega_0\sqrt{1-D^2}} \tag{8}$$

can be calculated for a 2nd order dynamic system, with stepwise excitation. From this, a stiffness

$$c_f = m_f\left(\frac{\pi - \arccos(D)}{t_{rt}\sqrt{1-D^2}}\right)^2 \tag{9}$$

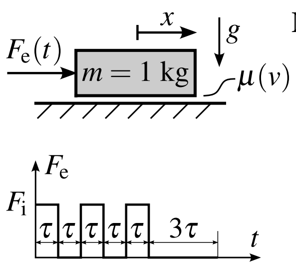
can be estimated using the natural frequency $\omega_0 = \sqrt{c_f/m_f}$.

In the case of dynamically oscillating frictional forces, the stiffness and damping ratio must be adapted accordingly to the respective system. For this purpose, reasonable parameters for t_{rt} and D must be selected. For a rise time t_{rt} significantly below the oscillation time of the acting force, a limited overshoot and oscillation of the stiffness force, a damping ratio of $0.3 < D < 0.7$ has proven itself.

According to Eq. (6) the stiffness coefficient c_f is determined by a fictitious displacement x_s, equivalent to the Adams and RecurDyn approach. In Eq. (9) the rise time t_{rt} of the stiffness force F_{c_f} is used to determine c_f.

4 Pulse Load

Some friction models, such as the standard regularization or the LuGre model, drift at pulse-like excitation as investigated by Rill et al. [5], among others. In order to examine the presented friction models for drift, the simple demonstration model of [5] is used. Figure 5 shows the demonstration model and its parameters. A series of three pulse-loads with an amplitude $F_i = 0.8F_s$ is applied to the mass. To make the different approaches comparable, it was necessary to increase the interval from $\tau = 0.1\,\text{s}$ to $\tau = 1.0\,\text{s}$. The friction models are simulated with the standard parameters of the mbs package, as shown in Fig. 5.



Model Parameter	**Adams and RecurDyn**	**Simpack**	**LuGre**
$\mu_s = 0.6$			$\sigma_0 = 10.0\ \frac{\text{N}}{\text{mm}}$
$\mu_d = 0.3$	$v_s = 0.10\ \frac{\text{mm}}{\text{s}}$	$c_f = 100 \cdot 10^3\ \frac{\text{N}}{\text{mm}}$	$\sigma_1 = 0.1\ \frac{\text{Ns}}{\text{mm}}$
$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$x_s = 0.01$ mm	$d_f = 10\ \frac{\text{Ns}}{\text{mm}}$	$\sigma_2 = 0.0\ \frac{\text{Ns}}{\text{mm}}$
$\tau = 1.0$ s			$v_a = 1.0\ \frac{\text{mm}}{\text{s}}$
$F_i = 4.7088$ N			$\alpha = 2.0$

Figure 5. Simple demonstration model with model parameters and standard friction model parameters of the corresponding mbs package

Fig. 6a shows that the LuGre model drifts as expected. The static friction force is not reached and the LuGre model breaks too early, see Fig. 6c. Fig. 6a also shows the result of regularization according to μ_s in RecurDyn. Due to $F_i > F_d$, the sliding friction level is exceeded and the mass starts to accelerate. With regularization to μ_s, only a very small drift occurs, which depends on the regularization velocity v_s. The regularization in Adams, which includes both μ_s and μ_d, shows the same result, because a STEP5 function is used for the transition to μ_s, also. In Fig. 6b it can be seen that the Simpack regularization absolutely drifts a little further ($\Delta x = 25.0\,\mu\text{m}$) than the STEP5 function in Adams or RecurDyn. This is due to the different slopes of the two regularizations. They are slightly higher for the STEP5 function than for the sin-function, as can be seen in Fig. 2. Therefore, the friction force is generated at a lower relative velocity.

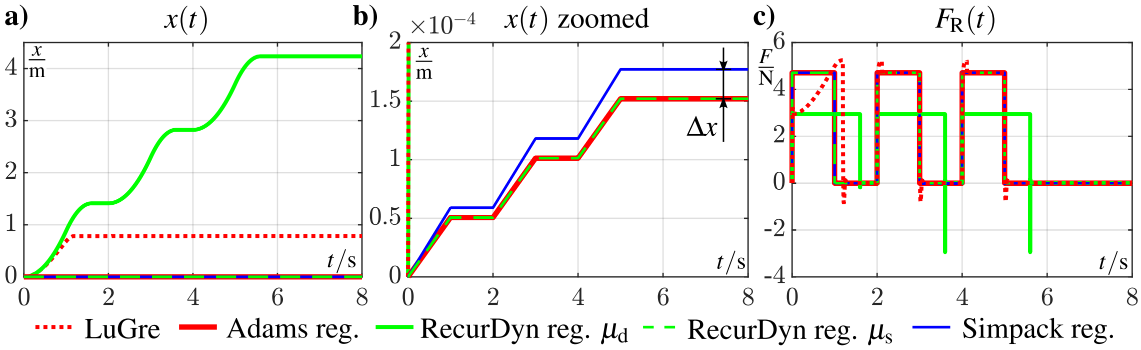


Figure 6. Simulation results for standard regularizations **a)** Displacement of LuGre model and RecurDyn (μ_d) **b)** Displacement of Adams, Simpack and RecurDyn (μ_s) **c)** Friction force

Fig. 7 shows the results of the stick-slip models. There is no drift in all three models. The displacement in Fig. 7a and the force curve in Fig. 7c show no difference between Adams and RecurDyn. The maximum displacement $x \approx 5.0\,\mu\text{m} < x_s$ is smaller than the regularization displacement. It can also be seen that this stick-slip model needs about 0.5 s to return to its initial position after no excitation. There is a significantly lower deflection of $x = 0.047\,\mu\text{m}$ with pulse-like excitation in Simpack, see Fig. 7b. The significantly lower displacement is due to the different standard parameters. Compared to Adams parameters, Simpack's stiffness results in a maximum displacement $x_s = \frac{F_s}{c_f} = 5.886 \cdot 10^{-5}$ mm at the static friction force. If the regularization parameter x_s is significantly reduced in Adams and RecurDyn, a very stiff characteristic curve is generated. This can lead to difficulties for the solver.

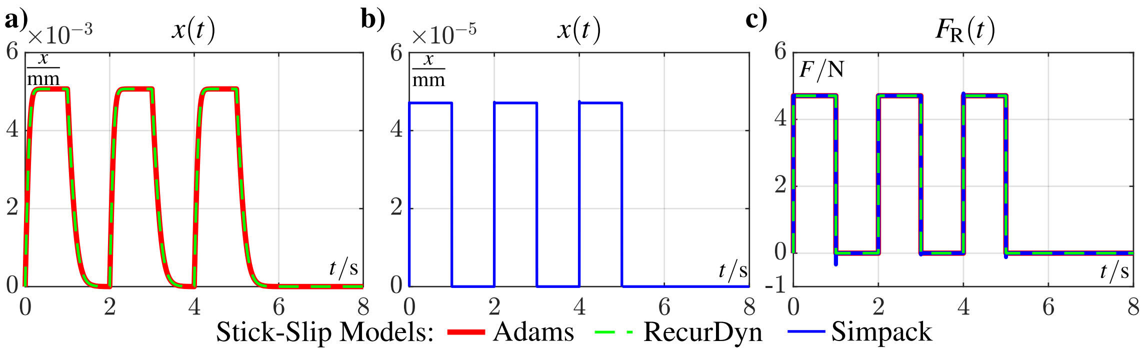


Figure 7. Simulation results for the stick-slip models **a)** Displacement of Adams and RecurDyn **b)** Displacement of Simpack **c)** Friction forces

5 CRANE FESTOON MODEL

The crane festoon model (Fig. 8) used by Rill et al. [5] is a useful practical application because it combines a variety of friction phenomena into a general multibody system. This model allows us to investigate breakaway behavior at different pulse loads as well as the stick-slip effect. Additionally, when cable trolleys are moved in positive and negative directions, the friction model must dynamically change the sign of the friction force as a result.

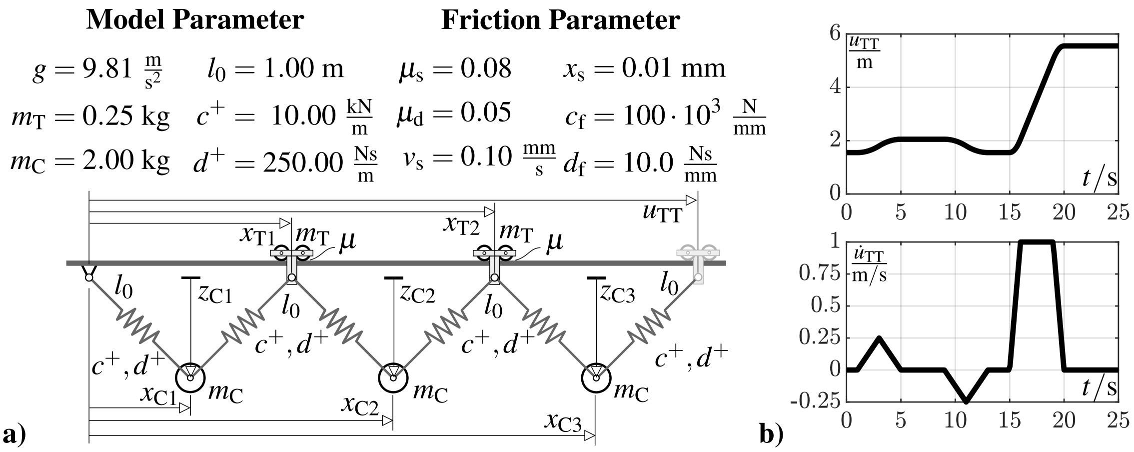


Figure 8. **a)** Crane Festoon model used by Rill et al. [5] and its parameters **b)** Excitation $u_{TT}(t)$ and $\dot{u}_{TT}(t)$

The model consists of a rail on which cable trolleys (T) can move in x-direction. The trolleys are connected by a cable, modeled by lumped masses (C1-C3) and spring-damper elements. The towing trolley is driven (rehonom) by the predefined function $u_{TT} = u_{TT}(t)$. The other trolleys are free in x-motion. The parameters l_0 defines the unloaded length, c^+ defines the stiffness and d^+ defines the damping of the cable.

Fig. 8b shows the motion specification and its time derivative. The first second ensures a quasi-static state. From 1 s to 15 s, the towing trolley is moved slowly to and fro. As a result, only trolley 2 is set in motion, but not trolley 1. After 15 s, a fast expansion occurs which leads to oscillations in the movement of the trolleys. The crane festoon model shown in Fig. 8 was used for the following studies.

5.1 Friction in General

The displacement and velocity of the trolleys and the frictional forces between the cable trolleys and the rail are each calculated by the corresponding mbs package. Only the specific stick-slip models of the packages will be discussed here. The LuGre model and a regularized friction model have already been discussed on the festoon model in detail in Rill et al. [5].

Figure 9 shows the simulation results of trolley 2 (T2). There is no significant difference between Adams and RecurDyn observed, only minor inconsistencies, which could be due to the different solvers. In addition, the position and velocity curve for Simpack does not show any significant differences to Adams and RecurDyn. In the transitions from sliding to sticking ($t \approx 4.5$ s, $t \approx 12.6$ s, $t \approx 19.9$ s, ...) the static friction peak is missing in Simpack. The sliding friction is modeled by the dynamic Coulomb friction force and switched to the static friction force by the condition $v = 0$.

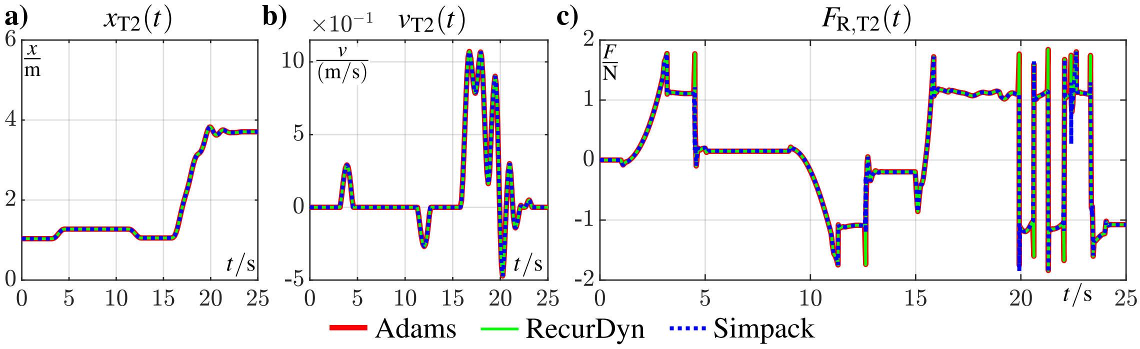


Figure 9. Trolley 2 results **a)** Displacement **b)** Velocity **c)** Friction force

5.2 Friction in Detail

Fig. 10a shows the time history of the friction coefficient $\mu(t) = \frac{F_R}{F_N}$ at the second transition from sliding to sticking ($t \approx 20.6$ s) at the extension maneuver. Adams and RecurDyn keep the regularization as defined and reach the maximum static friction value of $\mu_s = 0.08$. For Simpack, the maximum static friction value μ_s exceeds by $\Delta\mu = 0.0023$. According to the definition of the switching condition $F_{c_f} > F_s$ the actually acting friction $F_R = F_{c_f} + F_{d_f}$ exceeds the specification by the damping component.

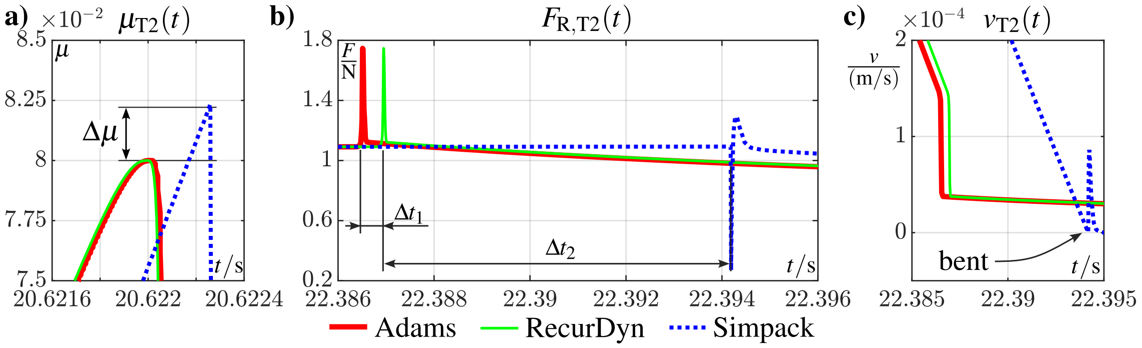


Figure 10. Detailed results of trolley 2 **a)** Dynamic overshoot at stick to slip **b)** Friction force at slip to stick transition **c)** Velocity at slip to stick transition

Fig. 10b shows the frictional force during the transition from sliding to sticking at $t \approx 22.39$ s. A time offset in the force curves of the three mbs packages can be seen. This is $\Delta t_1 = 0.4$ ms between Adams and RecurDyn and $\Delta t_2 = 7.3$ ms between RecurDyn and Simpack. The difference between Adams and RecurDyn may because by the different solvers applied as standard in the mbs packages. The qualitative behavior is comparable except for the time offset. The larger time offset to Simpack results on the one hand from the switching condition $v = 0$, which occurs slightly later than the peak of Adams and RecurDyn at $v = v_s$ (Fig. 10c) and on the other hand from the slightly higher friction accumulated in the system due to the overshoots. At $t = 22.3942$ s a step in the Simpack friction force can be seen. Before $t = 22.3942$ s, the Simpack model is in sliding mode. In the next time step, the switching condition $v = 0$ gets triggered and the model changes to sticking mode. The default setting "unloaded stiffness" sets the relative displacement $x = 0$ at this time. At the point of switching, the static friction force $F_R = c_f x + d_f v = 0$ and must therefore be built up first. Fig. 10c shows the velocity during the transition from sliding to sticking at $t \approx 22.39$ s. At $v = 0$ ($t = 22.3942$ s), a bent can be seen in the velocity curve of Simpack. This is caused by the step in the friction force curve. Since the step in the friction force curve is generated by the friction

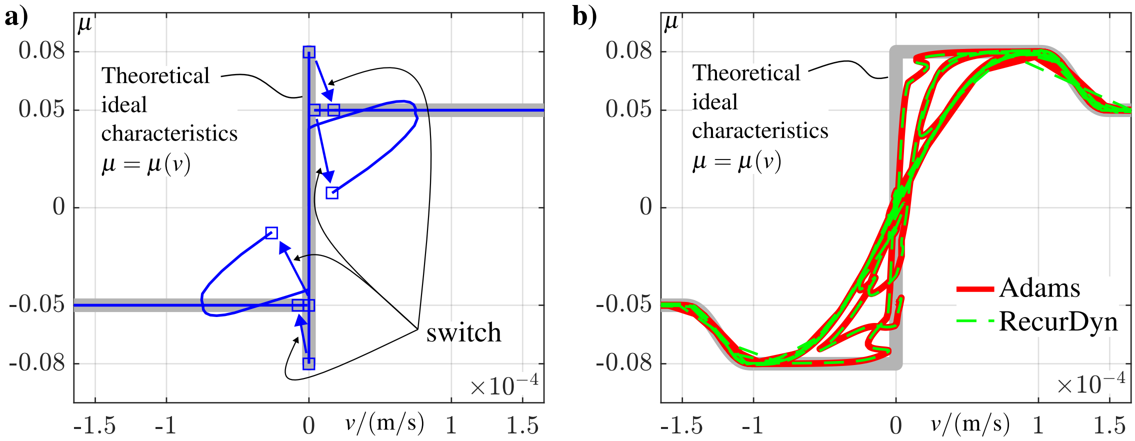


Figure 11. Friction characteristics $\mu(v)$ simulated with the festoon model **a)** Simpack stick-slip model **b)** Adams and RecurDyn stick-slip model

model and not by the actual force or velocity acting on trolley 2, a step occurs. That results in a bent in the velocity curve.

Fig. 11a shows the characteristics of the Simpack model. To improve the resolution, only the first 12.5 s were plotted. A square is used to represent the output values during switching, and no other values were calculated between these points. Ideally, switching would occur at $v = 0$. At $v \approx 0$ the friction force is $F_R = c_f x + d_f v = 0$ and for $|v| > 0$ the friction force is $F_R = -(v/|v|)\mu_d F_N$. Due to the resolution of $\Delta t_{step} = 5 \cdot 10^{-6}$ s the velocity jumps at the transition from $F_{R,stick}$ to $F_{R,slip}$ or from $F_{R,slip}$ and $F_{R,stick}$. At the transition from $F_{R,slip}$ to $F_{R,sick}$ the static friction force oscillates. This is therefore in the dissipative quadrants. In the first 12.5 s, the system transits from sticking to sliding in both positive and negative directions. During the slowly to and fro motion, the friction force is built quasi-statically, and no dynamic overshoot occurs. The static friction coefficient of $\mu = 0.08$ is maintained.

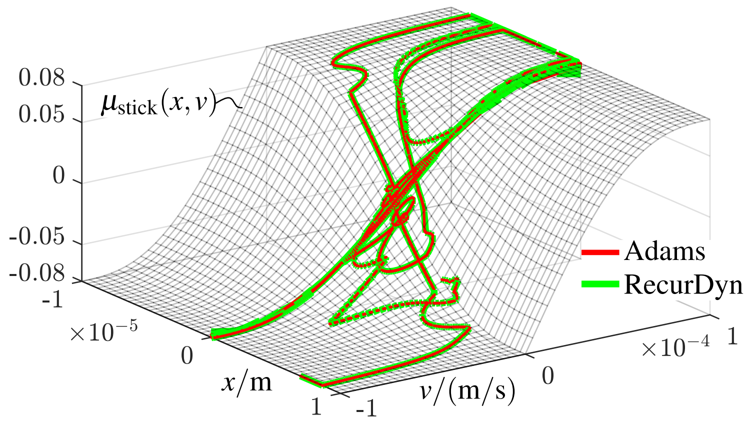


Figure 12. In detail, the coefficient of friction $\mu(x,v)$, as well as the stiction region $\mu_{stick}(x,v)$ of Adams and RecurDyn.

Fig. 11b shows the friction characteristics $\mu(v)$ of the stick-slip model from Adams and RecurDyn during the entire 25 s. The static friction coefficient μ_s is maintained. Ambiguities occur for $-v_s < v < v_s$ due to additional determination of μ by the displacement x. For $|v| > v_s$, the friction coefficient is transitioned to the dynamic friction coefficient μ_d by the STEP5 function. Fig. 12 shows the general stiction characteristics $\mu_{stick}(v,x)$ (Black) of Adams and RecurDyn and the simulated friction coefficients, respectively. Both the Simpack and the Adams or RecurDyn models deviate from the respective theoretically ideal friction characteristic $\mu = \mu(v)$, as seen in Fig. 1.

6 CONCLUSIONS

This paper clarifies and compares the standard friction models of Adams, RecurDyn and Simpack. First, the standard regularizations are considered. These regularize the ambiguous friction curve over velocity into a continuous curve. This has the disadvantage that no long-term stiction is possible. Since a relative velocity must be present to generate a friction force. Therefore, second, the focus is on the specific stick-slip models using two different approaches. Adams and RecurDyn enhance the standard regularization by a regularization over the displacement. Simpack models sticking by a spring-damper element and switches to dynamic Coulomb friction in sliding cases. Both approaches allow long-term stiction.

The comparison with the Festoon model shows no significant differences in the results. However, a closer look reveals minor specific differences. In both approaches, a function $F_R(x,v)$, which also takes displacement into account, replaces the friction characteristics $F_R(v)$, in the stiction region. As a consequence, the ideal friction characteristics $F_R(v)$ is not fulfilled in this region. Simpack does also not take the Stribeck effect into account.

From a practical point of view, these studies were carried out using the default values of the mbs packages. As these differ too much, a performance analysis was not carried out in this paper. An investigation of the runtime performance and a detailed consideration of the individual friction models will be the subject of further research and publications.

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