ECCOMAS Thematic Conference on Multibody Dynamics
July 24 - 28, 2023, Lisbon, Portugal

Testings of discrete modelings of planar compliant mechanisms through flexible multibody simulations

Orazio Sorgoná[1,3], Marco Cirelli[1,3], Matteo Verotti[2], Oliviero Giannini[1,3]

[1] Department of Industrial Engineering
Niccoló Cusano University
Via Don Carlo Gnocchi 3, 00166 Rome, Italy
[orazio.sorgona, marco.cirelli, oliviero.giannini]@unicusano.it

[2] DIME
University of Genova
Via all'Opera Pia, 15, 16145 Genoa, Italy
matteo.verotti@unige.it

[3] ATHENA
European University
Advanced Technology Higher
Education Network Alliance

ABSTRACT

Compliant mechanisms are continuous physical systems, gaining their mobility through the deformation of flexible parts of theirs. In the field of multibody simulations, compliant mechanisms can be modeled either by FEM in the full-flexible simulations, or by simpler modelings in the discrete-flexible. Some of these discrete modelings of compliance had been adopted and tested against the results of the full-flexible simulations, in the case of study of a complaint open-chain mechanism. Also, the linear theory of the ellipse of elasticity had been used. Results show that the discrete modelings describe well the cases of large displacements, and with a computation time reduced by two orders of magnitude with respect to the full-flexible simulations. The theory of the ellipse of elasticity is validated in the cases of small displacements.

Keywords: compliant mechanisms, flexible multibody simulations, nonlinear modeling, computational costs.

1 INTRODUCTION

Compliant mechanisms are physical systems which are continuous or made of integrated elements. Therefore, their mobility is gained through the load-dependent deformations of flexible zones, generally called flexures [1]. The absence of contacts and the usual monolithic structure of these systems imply minimization of friction, wear, backlash, lubrication, possible integration of functions, implementability in MEMS-based technologies and reduction of fabrication times and costs. Because of these profitable features, in the last decades compliant mechanisms has been implemented in a variety of applications, in the areas of precision machining and manufacturing [2], MEMS [3, 4], aerospace [5], surgery [6], vibrations isolation [7], robotics [8], power transmission [9], imprint lithography [10], positioning systems [11], and others. The increasing interest in the applications is confronted with the difficulties in the modeling and design of compliant mechanisms. In fact, both continuum and applied mechanics issues are to be taken into account, because in these systems the kinematic and elastomechanical problems are intrinsically coupled and cannot be studied in sequence, as for the rigid-linkages. To address the complexity of the problem, a wide range of models and design strategies has been developed, stemming from different approaches. Some comprehensive overviews of the state of the art of the analysis and synthesis procedures of compliant mechanisms can be found in [12, 13, 14]. More specifically, the modeling and design approaches can be broadly classified into the following groups.

a) Continuum-based, which start from considering the continuum structural nature of the systems. This approach comprises methods like FEM as a tool of analysis, and structural optimization, and generative design, as procedures of synthesis.

b) Kinematic, which, reversely, are based upon the required mechanical performances of the systems. This approach comprises methods like rigid body replacement as a tool of analysis, and the freedom-and-constraint-topology method (FACT), and rigid body replacement, as procedures of synthesis.

c) Building blocks, which can be seen as an intermediate way between the previous two. In fact, this approach is based on the kinetostatic characterization of primitive blocks, deriving from some continuum and/or kinematic approach. Eventually, the primitive blocks reduce to the single flexures. The whole compliant mechanism is then approached through network principles, in order to reduce to its component blocks in the analysis step, and to arrange the blocks in order to obtain the required global properties as procedures of synthesis.

In the field of multibody simulations compliant mechanisms can be modeled in several ways, similarly as to the different approaches [15]. More specifically, the continuum-based kind of modeling is called full-flexible, FFlex, and employs FEA computations to simulate the behavior of the flexures. The results obtained are accurate, nevertheless the simulations are time-consuming. Moreover, the results from analysis do not lead to an easy identification of significant parameters for the procedures of synthesis. For these reasons, it is profitable to work with simpler models that comprise a finite number of parameters, provided that the models are sufficiently accurate. The multibody implementations of discrete models of compliance are called discrete-flexible, DFlex, and can be classified as:

- rigid body replacements of the flexures by rigid linkages and lumped springs, it is by pseudo-rigid body models, PRBM [16, 17, 18, 19]; or
- characterization of an elastic suspension connecting two rigid links, i.e. "two-port" suspension, through compliance (or stiffness) matrix relations [20, 21, 22];

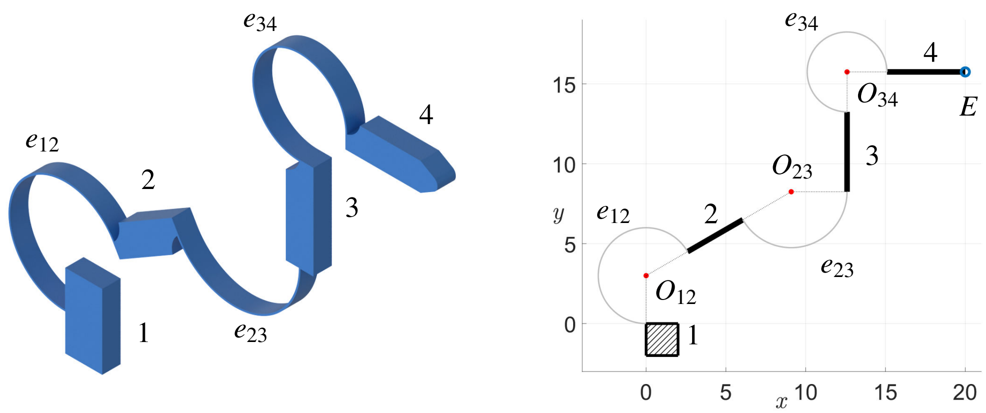
These models are introduced in Sec.3, together with the geometric model of the ellipse of elasticity. This model has been recently applied to the linear analysis of two-port planar suspensions characterized by open, closed, hybrid compliant kinematic chains [23].

The present study aims to test the accuracy of the discrete models of compliance, in the fields of both small and large displacements. Furthermore, the computational behavior of the DFlex and FFlex implementations are compared. A compliant open chain is taken as case of study, and modeled in multibody environment, according to the two aforementioned methodologies. In particular, as exposed in Section 3 the linear matrix characterization can be nevertheless implemented in a kinematic nonlinear modeling, in what is here called Sequential Compliance Matrix-Based Method, SCMBM. Moreover, the model of the ellipse of elasticity is applied for a the initial linear characterization of the mechanism under study.

2 Case of study

The compliant open kinematic chain shown in Fig. 1a is chosen as case study. From a topological point of view, open chains represent the simplest arrangement of flexures. On the other hand, they can exhibit a strong nonlinear behavior, even if their flexures undergo relatively small deformations. In particular, the compliant mechanism is composed of the rigid links 1, 2, 3, and 4, connected sequentially through the elastic suspensions e_{12}, e_{23}, e_{34}. The suspensions are the uniform circular arcs represented in Fig. 1b, whose geometric parameters are listed in Table 1. The in-plane thickness h of the flexures is 0.1 mm, whereas their out-of-plane dimension b is 1.0 mm. Body 1 represents the fixed frame, whereas point E of body 4 is the interaction point between the mechanism and the external environment. The analyses focus on the displacements and rotations of body 4 subsequent to the generalized forces applied to E.

It is worth noting that point E is however arbitrary, therefore the results can be taken as valid with no loss of generality.



(a) Open-chain compliant linkage (b) Geometric sizes and features.

Figure 1: Case of study of a compliant mechanism.

Table 1: Geometric parameters of the flexures: center coordinates, radius, and subtended angle.

Arc	O (mm, mm)	r (mm)	start angle (rad)	amplitude (rad)
e_{12}	(0.000, 3.000)	3.0	$-\pi/2$	$-4\pi/3$
e_{23}	(9.093, 8.250)	3.5	$-5\pi/6$	$5\pi/6$
e_{23}	(12.593, 15.750)	2.5	$-\pi/2$	$-3\pi/2$

3 Discrete modelings

In this Section, the discrete modelings of compliance adopted are introduced and exemplified with regard to the case of study. To simplify the subsequent exposition, some definitions pertaining to a two-port suspension, i.e. an elastic suspension between two rigid bodies, are firstly given. More specifically, one of the two connected bodies can be taken as the body upon which a system of active loads is applied, and therefore it can be termed as the *action body*. Accordingly, the system of loads on the other body can then be taken as reactive, and the body termed as the *base body*. Following the same reasonings, the relative displacements of the end bodies produced by the systems of loads are considered displacements *of* the action body, relatively to the base body.

3.1 Pseudo-Rigid Body Model

The Pseudo-Rigid Body Models, PRBM, belong to the rigid body replacement methods of analysis of compliant mechanisms. More specifically, these models replace a flexure with some rigid linkage and lumped springs associated to its kinematic pairs. Therefore, the compliant analysis is transformed into a standard kinematic analysis, in which the lumped springs account for the load-dependence of the displacements. In particular, a symmetric kinematic chain comprising three revolute joints had been developed as a PRBM for uniform circular arcs [19]. The model has been implemented in the case of study, according to the authors' indications on the sizing of the links lengths and the stiffnesses k_{θ_i} of the lumped springs, on the basis of the geometric and kinetostatic features of each arc. An example si given in Fig. 2.

3.2 Ellipse of elasticity

The ellipse of elasticity is a conic that can be associated to any planar two-port suspension, and therefore to any two-port planar compliant building block. The ellipse represents in a compact way the generic 3-DoF linear kinetostatic of the suspension, hence the elastic problem is transformed in

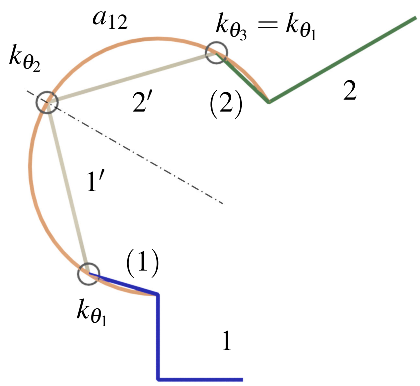


Figure 2: 3R pseudo-rigid body modeling of arc e_{12}.

a geometric problem with a straightforward solution. This is accomplished through antiprojective polarity transforms in the projective plane, and a scalar magnitude w associated to the geometry of the conic. The magnitude w is called "elastic weight", and has the physical meaning of a moment-rotation compliance, to work as a factor of scale between the magnitudes of loads and displacements. More specifically, $\theta = wM_C$, where M_C the moment of the resultant active load with regard to the ellipse center C, and θ is the rotation of the action body. The conic is the momental ellipse of the distribution in the suspension of the moment-rotation compliance, i.e. elastic weight. In fact, it's analogous to the ellipse of inertia, as the momental ellipse of the distribution of mass in a rigid body. In particular, for an inextensible Euler-Bernoulli beam with any section and axis geometry, the ellipse of elasticity can be defined by direct computation of the moments of the bending compliance $(EI)^{-1}$ over the beam axis, where E is the Young's modulus of the material, and I is the section moment of inertia. The local compliance can vary along the axis, whereas the expression EI must be substituted with the proper integral in the case of non-uniform stiffness on the section. If a flexure cannot be treated as an inextensible beam, the ellipse can anyway be defined through three linear quasi-static tests.

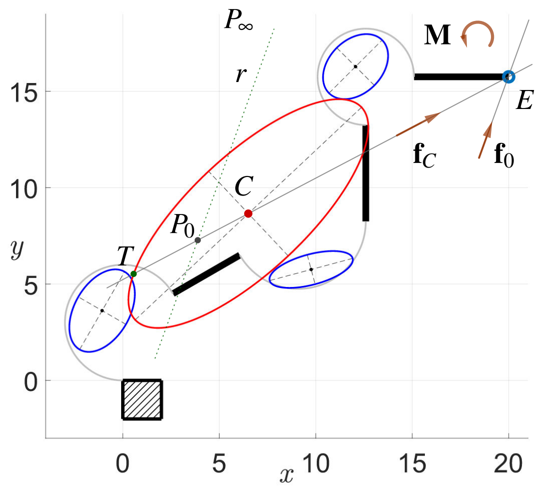


Figure 3: Ellipses of elasticty in the case of study

As a further step, the ellipse associated to a serial or parallel arrangement of suspensions can be defined on the basis of the components ellipses. Therefore, a serial, parallel, or mixed compliant mechanism can be characterized in this way. In particular, with regard to uniform circular arcs as in case of study, in the case of a with radius r and semi-amplitude γ, the elastic weight is computed immediately as $w = 2r\theta/EI$. The ellipse center C is located on the arc axis of symmetry, between the circle center O and the arc, at a distance $\overline{OC} = \frac{\sin(\gamma)}{\gamma} r$. The lesser ellipse semiaxis b lies along the axis of symmetry, whereas the greater semiaxis a is perpendicular to it. The semiaxes lengths are such that:

$$a^2 = \frac{r^2}{2}\left(1 - \frac{\sin(2\gamma)}{2\gamma}\right), \qquad b^2 = r^2 - \left(\overline{OC}^2 + a^2\right). \tag{1}$$

On the basis of the ellipses relative to the arcs it's then obtained the series ellipse characterizing body 4 with regard to the ground body 1. In Fig. 3 are shown the ellipses relative to the flexures and the resultant series ellipse with regard to the case of study. Some properties of the antiprojective polarity can be pointed out in the example. In particular, forces acting on E result in fields of displacements with poles upon line r, which is the antipolar line of point E with respect to the series ellipse $\mathscr{E}$. In particular, force $\mathbf{f}_0$ with line of action parallel to r produces rotations of body 4 about point P_0, which the intersection point of r and the line s trough E and the ellipse center C. It holds the relation: $\overline{CECP_0} = \overline{CT}^2$, where T is one of the two intersection points between line s and the ellipse. Further, line$r$ is parallel to the tangent line to the ellipse at point T. On the other hand, force $\mathbf{f}_C$ along line s produces a pure translation perpendicular to r, whereas a pure moment $\mathbf{M}$ applied to body 4 produces a rotation about the ellipse center C.

3.3 Sequential Compliance Matrix-Based Method

The linear kinetostatics of a two-port building block can be numerically expressed through a 6×6 matrix, reducing to 3×3 for the planar behavior. The matrix can be of compliance, or inversely of stiffness, and relates the generalized loads applied to the action body and reduced to some point M of its, and the displacements of M together with the magnitude of the rotations of the action body. It's noted here that this matrix differs from a finite-element matrix, which, besides, has doubled dimensions [12]. The definition of compliance and/or stiffness matrices with relation to the flexures serves as a basis for the linear modeling of a whole compliant mechanism. This comprises the Compliance Matrix Method, which develops accordingly to the same principles as for the ellipses of elasticity. It's out of the scope of this article a comparison between the two methods. The upperleft 2×2 block of a matrix $\mathbf{C}_M$ (or $\mathbf{K}_M$) characterizes the "single-point" conditions, i.e. of forces with lines of action passing through point M, and displacements of M itself. By diagonalizing the block, the two perpendicular directions of parallelism between forces and displacements are found [22]. These directions are also those of the greater and the lesser compliances (or stiffnesses) along two generic perpendicular directions.
Significantly, the matrix characterization can however be implemented in higher-orders modelings according to a chain-algorithm. More specifically, in the case study each flexure is modeled by a matrix relative to circular beams, the expression of which can be found in [20] . Each matrix

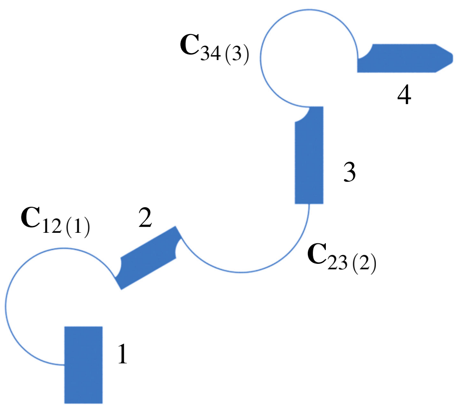


Figure 4: Example of application of SCMBM

is expressed in a reference frame that moves with the base body of the suspension to which the matrix is relative, as illustrated in Fig. 4. The matrix indices indicate the rigid links to which they are relative, and in parentheses the respective reference links. In this way, starting from the body 1 each matrix subsequent to the first accounts for all the previous displacements. This modeling has been implemented in [20], in the case of a full-inverse problem that takes as inputs relative displacements that are assigned, and givs as outputs the loads required to produce them, eventually applied to every moving link. In this case, the problem reduces to linear computations. In contrast, in the direct problem the solution of a system of trascendent equations is generally required. However, in the environments of multibody simulations the process of solution can be carried on in slim ways. This kind of modeling is here termed as Sequential Compliance Matrix-Based Method, SCMBM, by which a linear continuum model is embedded in a nonlinear kinematic model. In particular, a third-order model results in the case study.

3.4 Observation

It's worth noting that, in general, a PRBM is tailored on a particular kind of flexure, and usually requires complex procedures of optimization. Whereas, building block characterizations like the ellipse of elasticity and the matrix modeling can be simply defined on any kind of flexure and two-port suspension. In addition, these models are actually *separate* from the physical structure of the suspension, that is they represent functional features that can be embodied by continuos-numerous compliant systems. In fact, these methods can be employed with effectiveness in the procedures of *functional* synthesis of the compliant mechanisms, in which the definition of convenient kinetostatic features of the elements of the system is done aside of the definition of the structure. This can thus occur subsequently, or iteratively in a process of optimization.

4 Multibody simulations

In the environment of multibody simulations, both the described PRBM and the higher-order m model HMM have been implemented. Further, the FFlex simulations had been set up by FEM modeling of the flexures. As shown in Fig. 5, the geometry of the arcs allowed for a modeling through bidimensional elements, with a mesh of 20 elements per short side of the flexures. It has been hypothesed for the compliant mechanism a polymeric material with Young's modulus $E = 2.1$ GPa. Because the simulations were meant to be quasi-static, for each kind of modeling the parameters of density, mass, and damping, has been adjusted in order to minimize the dynamic behavior of the system. In the tests the displacements of body 4 are detected, under different load conditions. More specifically, forces applied to the end-effector E are considered, and a pure moment. This strategy has been explained in Section 2. Applied forces are non-follower, it is mantain the same direction as the input link 4 rotates. As outputs, the displacements of E from its starting position E_0 are detected, together with the global rotation of the body 4. In this way, the general fields of displacements can be appreciated, whereas eventual particular behaviors of point E subsequent to loads applied to it can be directly tracked.

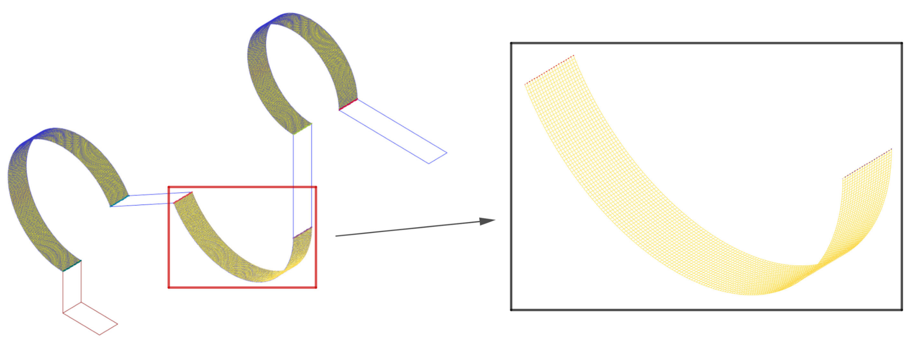


Figure 5: FFlex modeling of the compliant mechanism.

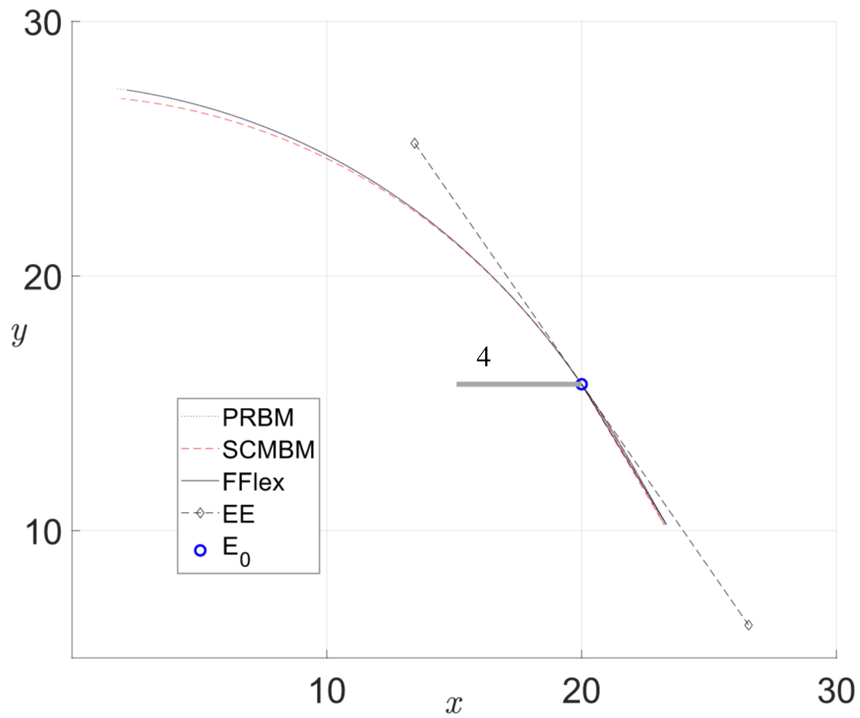


Figure 6: Horizontal forces applied on E (load case 1).

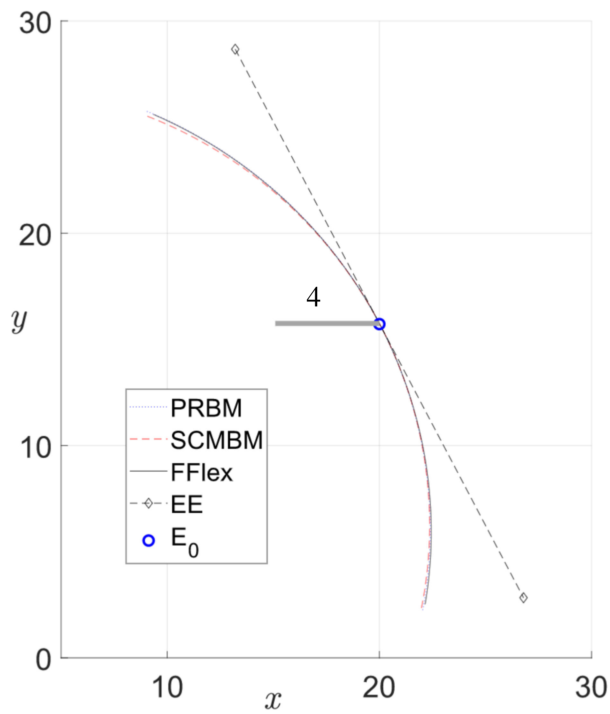


Figure 7: Pure moment applied to body 4 (load case 2).

4.1 Results

Results are presented for different load conditions. Forces are non-follower, that is don't change direction as body 4 rotates. In particular, Fig. 6 shows the paths of point E subsequent to forces applied to it, in the horizontal direction and in the two senses. Also, in Fig. 7 the paths are shown in the case of pure moments applied to body 4, in the two senses. The magnitudes of the rotations of body 4 that result from simulations differ at most for some percent unit, also in cases of rotations superior to one radiant. The discrete modelings implemented in the DFlex simulations show thus a very high accuracy, compared with the results of the Fflex. More specifically, it can be noted that the PRBM results in paths of E quite overlapped with those resulting from the continuum-modeling. However, the SCMBM doesn't show significant errors. Further cases are presented in Figs. 8 and 9, it is of forces applied to E in the directions of the greater single-point compliance and stiffness. These are estabilished on a linear modeling, as exposed in Section 3.3. Also in this cases, the Dflex simulations offer results very close to the Fflex ones. In particular, it can be noted in the case of grater stiffness (Fig. 9), where the magnitudes of applied loads are not the same in the two senses, that the systems exhibits a significant non-linear behavior. Also this behavior is very well described by the discrete modelings. The liner prediction by the theory of the ellipse of elasticity (EE) are validated for small displacements, as can be appreciated by the accuracy of the predicted tangent line at E_0. With regard to computation times, in each load case the DFlex simulations have required from 1 to 2 seconds about, to be performed through the machine and the software used. Whereas, the simulations with the Fflex modeling have required more than 6 minutes in each case.

5 Conclusions

The results of the simulations show remarkable agreements between the discrete and the full-flexible modelings, also in cases of strongly nonlinear behaviors. The model of ellipse of elasticity is validated in cases of small displacements. Moreover, in the conducted simulations the DFlex implementations had required a computation time reduced by two orders of magnitude. As small errors at the level of open chains propagate linearly and predictably in more complex arrangements, the results hold also at the level of a complex mechanism.The validated accuracy of the discrete modelings, and their major computational advantage, shows them profitable to be employed in flexible multibody simulations and in the modeling of compliant mechanisms.

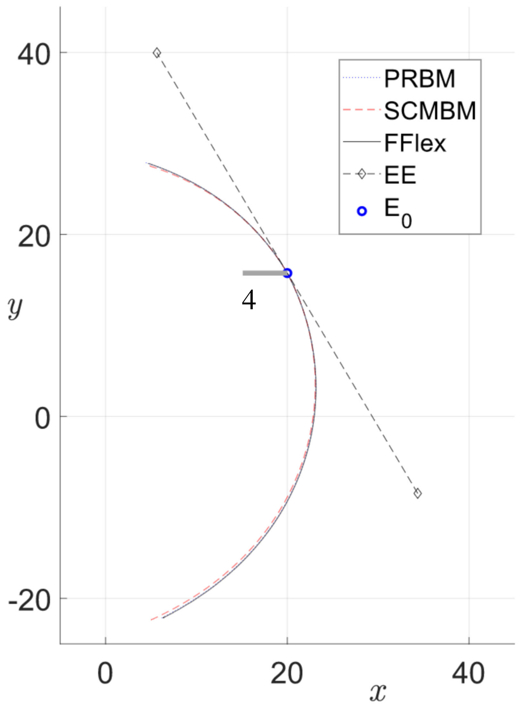


Figure 8: Applied forces on E on the direction of the greater linear compliance (load case 3).

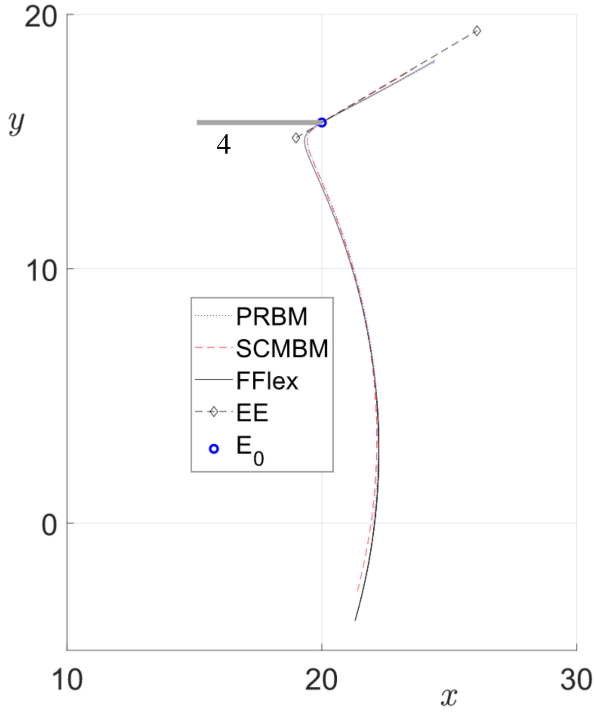


Figure 9: Applied forces on E on the direction of the greater linear stiffness (load case 4).

REFERENCES

[1] Howell, L.: Compliant Mechanisms. Wiley, New York NY (2001) ISBN: 978-0-471-38478-6.

[2] Tian, Y., Zhang, D., Shirinzadeh, B.: Dynamic modelling of a flexure-based mechanism for ultra-precision grinding operation. Precision Engineering **35**(4) (2011) 554–565 doi:10.1016/j.precisioneng.2011.03.001.

[3] Hao, G., Zhu, J.: Design of a monolithic double-slider based compliant gripper with large displacement and anti-buckling ability. Micromachines **10**(10) (2019) doi:10.3390/mi10100665.

[4] Verotti, M., Dochshanov, A., Belfiore, N.P.: A comprehensive survey on microgrippers design: Mechanical structure. Journal of Mechanical Design **139**(6) (may 2017) doi:10.1115/1.4036351.

[5] Zeng, W., Gao, F., Jiang, H., Huang, C., Liu, J., Li, H.: Design and analysis of a compliant variable-diameter mechanism used in variable-diameter wheels for lunar rover. Mechanism and Machine Theory **125** (2018) 240–258 doi:10.1016/j.mechmachtheory.2018.03.003.

[6] Thomas, T.L., Kalpathy Venkiteswaran, V., Ananthasuresh, G., Misra, S.: Surgical applications of compliant mechanisms: A review. Journal of mechanisms and robotics **13**(2) (2021) 020801 doi:10.1115/1.4049491.

[7] Valentini, P.P., Cirelli, M., Di Donato, S.: The compliant centrifugal pendulum as the vibration absorber with second-order elasto-kinematic approximation. Journal of Vibration and Control **11-12**(6) (2021) 1370–1381 doi:10.1177/1077546320941712.

[8] Morales Bieze, T., Kruszewski, A., Carrez, B., Duriez, C.: Design, implementation, and control of a deformable manipulator robot based on a compliant spine. The International Journal of Robotics Research **39**(14) (2020) 1604–1619 doi:10.1177/0278364920910487.

[9] Sorgonà, O., Giannini, O., Cirelli, M., Valentini, P.P.: Multibody simulations of a distributed-compliance helical transmission joint for largely misaligned shafts. In: Materials Research Proceedings, Vol. 26, St. Louis, MO, USA (2023) 671–676 doi:10.21741/9781644902431-108.

[10] Xu, K., Luo, H., Qin, J., Yang, M., Guo, S., Wang, L.: Flexible devices fabricated by a plate-to-roll nanoimprint lithography system. Nanotechnology **30**(7) (2018) 075301 doi:10.1088/1361-6528/aaf26f.

[11] Wang, T., Li, Y., Zhang, Y., Lin, R., Qian, J., Dou, Z.: Design of a flexure-based parallel xy micropositioning stage with millimeter workspace and high bandwidth. Sensors and Actuators A: Physical (2021) 112899 doi:10.1016/j.sna.2021.112899.

[12] Ling, M., Howell, L.L., Cao, J., Chen, G.: Kinetostatic and dynamic modeling of flexure-based compliant mechanisms: A survey. Applied Mechanics Reviews **72**(3) (01 2020) 030802 doi:10.1115/1.4045679.

[13] Henning, S., Linß, S., Gräser, P., Theska, R., Zentner, L.: Non-linear analytical modeling of planar compliant mechanisms. Mechanism and Machine Theory **155** (2021) 104067 doi:10.1016/j.mechmachtheory.2020.104067.

[14] Gallego, J.A., Herder, J.: Synthesis methods in compliant mechanisms: An overview. In: ASME Paper No. DETC2009-86845. (2009)

[15] Wasfy, T.M., Noor, A.K.: Computational strategies for flexible multibody systems. ASME. J. Mech. Des. **56**(6) (2003) 553–613 doi:10.1115/1.1590354.

[16] Cera, M., Cirelli, M., Colaiacovo, L., Valentini, P.P.: Second-order approximation pseudo-rigid model of circular arc flexure hinge. Mechanism and Machine Theory **175** (2022) 104963 doi:10.1016/j.mechmachtheory.2022.104963.

[17] Verotti, M.: A pseudo-rigid body model based on finite displacements and strain energy. Mechanism and Machine Theory **149** (2020) 103811 doi:10.1016/j.mechmachtheory.2020.103811.

[18] Valentini, P.P., Cirelli, M., Pennestrì, E.: Second-order approximation pseudo-rigid model of flexure hinge with parabolic variable thickness. Mechanism and Machine Theory **136** (2019) 178–189 doi:10.1016/j.mechmachtheory.2019.03.006.

[19] Venkiteswaran, V.K., Su, H.: A versatile 3r pseudo-rigid-body model for initially curved and straight compliant beams of uniform cross section. ASME. J. Mech. Des. **140**(9) (2018) 092305 doi:10.1115/1.4040628.

[20] Belfiore, N.P., Simeone, P.: Inverse kinetostatic analysis of compliant four-bar linkages. Mechanism and Machine Theory **69** (2013) 350–372 doi:10.1016/j.mechmachtheory.2013.06.008.

[21] Tang, H., Li, Y.: Design, analysis, and test of a novel 2-dof nanopositioning system driven by dual mode. IEEE Transactions on Robotics **29**(3) (2013) 650–662 doi:10.1109/TRO.2013.2248536.

[22] Krishnan, G., Kim, C., Kota, S.: An intrinsic geometric framework for the building block synthesis of single point compliant mechanisms. ASME. J. Mechanisms Robotics **3**(1) (2011) 011001 doi:10.1115/1.4002513.

[23] Sorgonà, O., Belfiore, N., Giannini, O., Verotti, M.: Application of the ellipse of elasticity theory to the functional analysis of planar compliant mechanisms. Mechanism and Machine Theory **184** (2023) 105308 doi:10.1016/j.mechmachtheory.2023.105308.