# Design of Fixed-Sequence Planar 5R Parallel manipulators with Adjustable Links

### Vigen Arakelian<sup>1,2</sup>

<sup>1</sup> Mecaproce / INSA-Rennes 20 av. des Buttes de Coesmes CS 70839, F-35708 Rennes, France vigen.arakelyan@insa-rennes.fr <sup>2</sup> LS2N-ECN UMR 6004
1 rue de la Noë, BP 92101
F-44321 Nantes, France
vigen.arakelyan@ls2n.fr

## ABSTRACT

The problem of moving a gripper through a path ensuring only its predetermined initial and final positions is a problem often encountered in various automation tasks. In other words, achieving the objective of moving the gripper along a non-imposed trajectory between two given positions, with the ability to change these positions periodically. One solution to the problem is to create fixed sequence manipulators with adjustable parts. The merit of the solution is that manipulation systems of this type with cyclic control have only one actuator. Note that the cyclic control system improves the operational reliability of the manipulation system and greatly reduces the cost of it. The present study proposes a new concept for the design of fixed-sequence planar 5R symmetrical parallel manipulators. The aim of the proposed design principle is that the two input links of the manipulator are interconnected. This interconnection is carried out via a four-bar linkage with adjustable links. The link lengths are determined in such a way that the initial and final positions of the gripper are ensured. Numerical simulations are performed to illustrate the proposed design concept.

**Keywords:** Design, Fixed-sequence manipulators, Planar 5R parallel mechanism, Geometric synthesis, Adjustable Links

## **1 INTRODUCTION**

The problem of the synthesis of the manipulation systems by the predetermined positions of the gripper can be solved by two ways: 1) by using robot manipulators with several degrees of freedom allowing precise reproduction of different positions of the gripper. In this case, the initial and final positions can be modified by the choice of the input parameters for each degree of freedom; 2) by synthesis of a mechanism with one degree of freedom allowing precise reproduction of a limited number of predetermined gripper positions. In this case, when modifying the initial and final position of the gripper, it is essential to adjust certain parameters of the mechanical system. The second method is considered to be the most optimum from the point of view of simplified control and minimum energy expenditure. It widens the field of application of fixed-sequence manipulators and permits to use efficiently the theory of geometrical kinematics applied to mechanism synthesis.

Different approaches and solutions devoted to this problem have been developed and documented [1], [2]. At the Institut de Productique de Besançon, to solve this problem, the Bennett mechanism (Fig. 1) was used [3], [4]. The suggested design concept provides the predetermined trajectory of the manipulated object through three predetermined positions of the gripper. The choice of three positions of the manipulated object is motivated by the fact that they correspond to three processing steps: loading of blank, machine working and removal of finished part. Such gripping

movements can be observed quite often during automation tasks. The problem relating to precise reproduction of three and four positions of the gripper was also treated by using RRRSR spatial mechanism [5]. It was suggested to use approximation synthesis methods [6]-[8] for the design of spatial fixed-sequence manipulation systems reproducing the spatial trajectories given by N positions of the gripper.

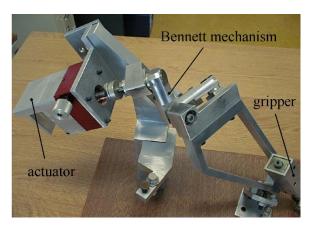


Figure 1. Fixed-sequence manipulator with Bennett mechanism.

An adjustable gear transmission system was also used to the modified planar 5R manipulator [9]. These gears of the transmission system provided the required rotation angles for two links mounted on the frame.

The present paper proposes a new design concept of fixed-sequence planar 5R symmetrical parallel manipulators. It is made possible by converting the planar 5R manipulator into a four-bar linkage with adjustable links and the application of the methods of geometric synthesis.

## 2 STATEMENT OF THE PROBLEM

Figure 2 is illustrates a kinematic scheme of the planar 5R parallel manipulator. The output axis P(x,y), which corresponds to the axis of the gripper, is connected to the base by two legs, each of which consists of three revolute joints and two links. The two legs are connected to a common axis P with the common revolute joint at the end of each leg. In each of the two legs, the revolute joint connected to the base is actuated. Such a manipulator can position the gripper freely in a plane xOy.

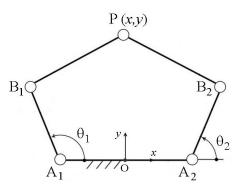


Figure 2. Planar 5R parallel manipulator.

In the given planar 5R symmetrical parallel mechanism each actuated joint is denoted as  $A_i(i = 1,2)$  the other end of each actuated link is denoted as  $B_i(i = 1,2)$  and the common joint of the two legs is denoted as P, which is also the axis of the gripper. A fixed global reference system Oxy is located at the center of  $A_1A_2$  with the y-axis normal to  $A_1A_2$  and the x-axis directed along  $A_1A_2$ . The lengths of links are denoted as  $l_1 = A_1B_1 = A_2B_2$ ,  $l_2 = B_1P = B_2P$  and  $l_0 = OA_1 = OA_2$ . Now, let us consider that it would be necessary to handle a payload ensuring only two prescribed positions: initial and final. That is, the payload's path of motion is not imposed and may be an arbitrary curve. This is often the case with many tasks of picking up the payload and placing

it in another location. Such a task can also be repetitive within a specified time limit. Certainly, it can be achieved via a planar 5R manipulator with two degrees of freedom. However, given the conditions described, it is more reasonable to transform the 5R parallel manipulator into a mechanical system with one degree of freedom. For this purpose, an adjustable transmission system will be added to modify the initial structure of the manipulator (Figure 3). The added link  $C_1C_2$  will form a four-bar mechanism providing the required rotation angles  $\theta_1$  and  $\theta_2$  for input links. The suggested methodology is considered optimal in terms of its simplified control and minimum energy expenditure.

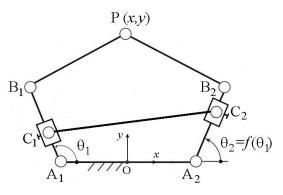


Figure 3. Fixed-sequence planar 5R parallel manipulator.

#### **3** SYNTHESIS OF FIXED-SEQUENCE MANIPULATORS WITH ADJUSTABLE LINKS BY TWO GIVEN POSITIONS OF THE GRIPPER

Thus, if the initial  $x_i$ ,  $y_i$  and the final  $x_f$ ,  $y_f$  positions of output axis P(x, y) are known, the initial and final values  $\theta_{1i}$ ,  $\theta_{1f}$  and  $\theta_{2i}$ ,  $\theta_{2f}$  of the angles  $\theta_1$  and  $\theta_2$  can be obtained from the inverse kinematics [10].

$$\theta_{jk} = 2\tan^{-1}(z_{ik})_2 j = 1,2, \ k = i, f$$

where

$$z_{jk} = \frac{-b_{jk} + \sigma_i \sqrt{b_{jk}^2 - 4a_{jk}c_{jk}}}{2a_{jk}}$$

$$a_{1k} = l_1^2 + y_k^2 + (x_k + l_0)^2 - l_2^2 + 2(x_k + l_0)l_1$$
  

$$b_{1k} = -4y_k l_1$$
  

$$c_{1k} = l_1^2 + y_k^2 + (x_k + l_0)^2 - l_2^2 - 2(x_k + l_0)l_1$$
  

$$a_{2k} = l_1^2 + y_k^2 + (x_k - l_0)^2 - l_2^2 + 2(x_k - l_0)l_1$$
  

$$b_{2k} = b_{1k} = -4y_k l_1$$
  

$$c_{2k} = l_1^2 + y_k^2 + (x_k - l_0)^2 - l_2^2 - 2(x_k - l_0)l_1$$

with  $\sigma_1 = 1$ ,  $\sigma_2 = -1$  corresponding to the configuration of the manipulator shown in Fig. 3. Then, knowing the input angles  $\theta_1$  and  $\theta_2$ , the position of the output axis P(x, y) of the manipulator is determined via forward kinematics [10]. The synthesis of the linkage  $A_1C_1C_2A_2$  can be achieved by determining the lengths of the links  $A_1C_1$  and  $A_2C_2$ :  $l_{A_1C_1} = L_2$  and  $l_{A_2C_2} = L_4$ , taking into account that the nondimensional lengths of the links  $C_1C_2$  and  $A_1A_2$ :  $l_{C_1C_2} = L_3$  and  $l_{A_1A_2} = L_1 = 1$  are known.

By formulating the conditions of the geometric synthesis of the linkage  $A_1C_1C_2 A_2$ , the unknown length  $l_{A_1C_1} = L_2$  can be obtained from the polynomial equation:

$$a_4L_2^4 + a_3L_2^3 + a_2L_2^2 + a_1L_2 + a_0 = 0,$$

with

$$a_{4} = \beta_{3}^{2}$$

$$a_{3} = 2(\beta_{2}\beta_{3} - \alpha_{3}\beta_{1}\beta_{3} - \alpha_{1}\beta_{3}^{2})$$

$$a_{2} = -\beta_{3}^{2}L_{3}^{2} + \beta_{1}^{2} + \beta_{2}^{2} - 2\alpha_{3}\beta_{1}\beta_{2} + 2\alpha_{2}\beta_{1}\beta_{3} - 4\alpha_{1}\beta_{2}\beta_{3} + \beta_{3}^{2}$$

$$a_{1} = -2(\beta_{2}\beta_{3}L_{3}^{2} + \alpha_{2}\beta_{1}\beta_{2} - \alpha_{1}\beta_{2}^{2} + \beta_{2}\beta_{3})$$

$$\alpha_{0} = \beta_{2}^{2}(1 - L_{3}^{2})$$
where,  

$$\alpha_{1} = \cos\theta_{1i}$$

$$\alpha_{2} = \cos\theta_{2i}$$

$$\alpha_{3} = \cos(\theta_{2i} - \theta_{1i})$$

$$\beta_{1} = \cos\theta_{1i} - \cos\theta_{1f}$$

$$\beta_{2} = \cos\theta_{2i} - \cos\theta_{2f}$$

$$\beta_{3} = \cos(\theta_{2f} - \theta_{1f}) - \cos(\theta_{2i} - \theta_{1i})$$

Then, the unknown length  $l_{\Delta C} = L_4$  will be determined:

$$L_4 = \beta_1 L_2 / (\beta_2 + \beta_3 L_2).$$

#### 4 ILLUSTRATIVE EXAMPLE AND SIMULATION RESULTS

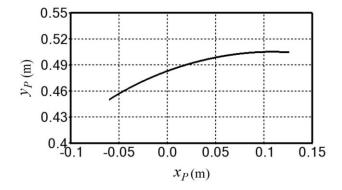
To illustrate the suggested design concept, the following geometric parameters of the planar 5R manipulator have been used:  $l_{A_1B_1} = l_{A_2B_2} = 0.36$  m;  $l_{B_1P} = l_{B_2P} = 0.3$  m;  $l_{OA_1} = l_{OA_2} = 0.24$  m. The trajectory of the gripper *P* is given by the initial position  $P_i$  with the coordinates  $x_i = -0.06$  m,  $y_i = 0.45$  m and the final position  $P_f$  with the coordinates  $x_f = 0.126$  m,  $y_f = 0.504$  m. From these values, the initial and final positions of the angles  $\theta_1$  and  $\theta_2$  are calculated:  $\theta_{1i} = 1.856$  rad.,  $\theta_{1f} = 1.246$  rad.,  $\theta_{2i} = 1.609$  rad.,  $\theta_{2f} = 1.191$  rad.

For the given values  $l_{A_1A_2} = L_1 = 1(l_1 = 0.48 \text{ m})$  and  $l_{C_1C_2} = L_3 = 1.05(l_3 = 0.504 \text{ m})$ , the following polynomial equation was obtained:

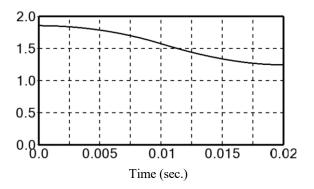
 $0.00081884L_2^4 + 0.01035443L_2^3 + 0.03931921L_2^2 + 0.07726310L_2 - 0.01711012 = 0.$ 

From which the length of the link  $A_1C_1$  is determined:  $L_2 = 0.2$  ( $l_2 = 0.096$  m) and then from  $L_4 = \beta_1 L_2/(\beta_2 + \beta_3 L_2)$ , the length of the link  $A_2C_2$ :  $L_4 = 0.298$  ( $l_4 = 0.143$  m).

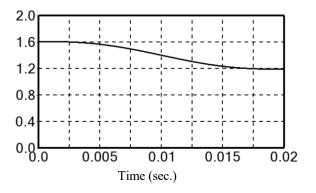
To validate the numerical example, simulations have been carried out on the software ADAMS. By these simulations, it was approved that the developed manipulator ensures the given initial and final positions of the gripper. The trajectory of the gripper, the variations of the angles  $\theta_1$  and  $\theta_2$  are shown in figures 4, 5 and 6. It is obvious that when changing the initial and final positions of the gripper, the lengths of the links  $A_1C_1$  and  $A_2C_2$  must be adjusted.



*Figure 4. Trajectory of the gripper*  $y_P = f(x_P)$ *.* 



*Figure 5.* Variations of the angle  $\theta_1$  (rad).



*Figure 6.* Variations of the angle  $\theta_2$  (rad).

#### **5 DISCUSSION**

The author believes that the proposed solution is practical, feasible and cost-effective, and applications for it can be found in areas requiring fast manipulation. The examined problem can also be solved for three given positions of the gripper, i.e. adding an intermediate position between the initial  $x_i, y_i$  and the final  $x_f, y_f$  positions. In this case, the length of the link  $C_1C_2$  will also be adjustable. However, it should be noted that the mathematical solution in this case will be much easier, since it is necessary to solve a system of linear equations and to find the parameters  $l_{A_1C_1} = L_2, l_{C_1C_2} = L_3$  and  $l_{A_2C_2} = L_4$ .

#### 6 CONCLUSIONS

This work tackles the problem of designing fixed-sequence planar 5R parallel manipulators to reproduce the predetermined initial and final positions of the gripper. It demonstrates how the manipulator can be designed with two prescribed positions of the gripper. For this purpose, the basic manipulator, which represents a parallel mechanism with two degrees of freedom, is transformed into a mechanism with one degree of freedom. An adjustable transmission system, which is a four-bar linkage, was added to the modified 5R manipulator. The link lengths of the created four-link linkage were determined by means of a geometric synthesis for two given positions of the input and output links. These two positions of the four-bar linkage were determined by means of the gripper. The formulation of the problem of geometric synthesis led to the solution of a fourth-degree polynomial. To illustrate the suggested design concept, simulations have been carried out via ADAMS software. The suggested methodology is considered optimal in terms of its simplified control and minimum energy expenditure. The merit of the solution is that manipulation systems of this type with cyclic control have only one actuator. Note that the control system improves the operational reliability of the manipulation system and greatly reduces the cost of it.

#### REFERENCES

[1] Sarkissyan Y.L. et al.: Synthesis of mechanisms with variable structure and geometry for reconfigurable manipulation systems. In: ASME/IFToMM International Conference on Reconfigurable Mechanisms and Robots. London, United Kingdom (2009) 195-199.

[2] Lee W.-T., Russell K., Shen Q., Sodhi R. S.: On adjustable spherical four-bar motion generation for expanded prescribed positions. Mech. and Mach. Theory 44 (2009) 247-254.

[3] Bennett G.T.: A new mechanism, Engineering 76 (1903) 777-778.

[4] Dahan M., Daha C., Lexcellent C.: Propriétés et utilisation du mécanisme de Bennett. Mech. and Mach. Theory 20 (1985) 189-197.

[5] Arakelian V. et al., Manipulator. Patent SU 1364467, January 7, 1988.

[6] Bottema O., Roth B.: Theoretical kinematics, New York: North-Holland Pub. (1979) 558p.

[7] Sarkissyan Y.L.: Approximation synthesis of mechanisms, Moscow: Naouka (1982) 304p.

[8] Arakelian V., Dahan M.: Design of fixed-sequence manipulator based on the generation of spatial coupler curves. In: Theory and Practice of Robots and Manipulators. Springer (2000) 147-154.

[9] Arakelian V.: Design of fixed-sequence planar 5R symmetrical parallel manipulators. International Journal of Mechanics 16 (2022) 91-97.

[10]Liu X.-J. et al.: Kinematics, singularity and workspace of planar 5R symmetrical parallel mechanisms. Mech. and Mach. Theory 41 (2006) 145–169.