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Rheonomic Cone Complementarity Constraints for the Efficient Simulation of Sliding and Vibrating Feeders with Non-Smooth Frictional Contacts

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ABSTRACT

The simulation of feeders and conveyors in the context of multibody dynamics is typically limited by the small time step needed to fully capture their interaction with a large number of transported bodies. In the present paper, we propose an efficient algorithm based on non-smooth rheonomic frictional contact constraints that can model the overall material drifting onto a stationary object with few parameters – namely, the eigenvalues of the 3D motion imposed by the feeder. The method can be effectively used to simulate, with reduced computational burden and large time steps, both vibrating feeders and traditional sliding conveyors. Comparative results from the proposed method and conventional full simulation of the feeder are presented, showing markedly superior performance while retaining overall system behavior.

Keywords: Rheonomic constraints, Nonsmooth Dynamics, Cone Complementarity Problem, Contacts, Feeders.

1 INTRODUCTION

Sliding and vibratory feeders are the most common industrial devices to relocate – and potentially orient – large quantities of small parts along a track. The transportation process is often combined with suitable machinery that allows to rotate these components or select only the ones having a desired orientation. The design of such feeders, with particular reference to bowl vibratory ones, is usually performed by trial and error, since the transportation process is complex to model in closed form and the dynamics of repeated impacts can be chaotic (see, for example [1]). Available analytical solutions make use of considerable simplifying assumptions, e.g. reducing the problem to a single degree-of-freedom vibrating system and modeling material as lumped masses that move on an inclined plane [2, 3, 4]. In order to overcome these restrictions, it is useful to restate the governing equations of the system in the more general framework of multibody dynamics. In this context, it is possible to simulate the motion of many components subjected to collision against an arbitrarily-moving conveyor. However, in the case of high-frequency vibrations or large number of parts, there is the need of reducing the simulation time step, potentially up to a point where required CPU time becomes prohibitive. The method proposed in this work addresses the exposed problem through an efficient and unifying approach that makes use of rheonomic cone complementarity constraints applied only during the contact phase of interacting bodies.

The rest of the paper is divided as follows. Section 2 introduces the notation and the mathematical background useful to model generic multibody systems subject to bilateral and unilateral constraints with friction. Section 3 illustrates the idea of the proposed method and describes its implementation in the context of Measure Differential Inclusion formulation. Section 4 is dedicated to test the proposed algorithm in comparison to conventional full simulation of some systems. Finally, Section 5 presents the evidences resulted by the work.

2 PROBLEM DESCRIPTION

The dynamic model of a generic multibody system subject to both bilateral and frictional unilateral constraints may be formulated as a Differential Variational Inequality (DVI) [5]

$$\mathbf{M}(\mathbf{q})\frac{d\mathbf{v}}{dt} = \widehat{\mathbf{f}}_t(\mathbf{q},\mathbf{v},t) + \sum_{i\in\mathscr{G}_A}\left(\widehat{\gamma}_n^i\mathbf{d}_n^i + \widehat{\gamma}_u^i\mathbf{d}_u^i + \widehat{\gamma}_v^i\mathbf{d}_v^i\right) + \sum_{i\in\mathscr{G}_B}\widehat{\gamma}_B^i\nabla\boldsymbol{\psi}^i \tag{1a}$$

$$\boldsymbol{\psi}^i(\mathbf{q},t) = 0, \quad i\in\mathscr{G}_B \tag{1b}$$

$$\widehat{\gamma}_n^i \geq 0 \perp \phi^i(\mathbf{q}) \geq 0, \quad i\in\mathscr{G}_A \tag{1c}$$

$$(\widehat{\gamma}_u^i,\widehat{\gamma}_v^i) = \underset{\mu\widehat{\gamma}_n^i \geq \sqrt{(\widehat{\gamma}_u^i)^2+(\widehat{\gamma}_v^i)^2}}{\operatorname{argmin}} \mathbf{v}^T(\widehat{\gamma}_u^i\mathbf{d}_u^i + \widehat{\gamma}_v^i\mathbf{d}_v^i), \quad i\in\mathscr{G}_A \tag{1d}$$

$$\dot{\mathbf{q}} = \Gamma(\mathbf{q})\mathbf{v} \tag{1e}$$

where: Eq. (1a) represents the equation of dynamics in terms of generalized coordinates $\mathbf{q}$, generalized velocities $\mathbf{v}$, block-diagonal mass matrix $\mathbf{M}$, total external forces $\widehat{\mathbf{f}}_t$ (including centrifugal and gyroscopic terms), set of unilateral constraints forces $\mathscr{G}_A$, and set of bilateral constraint forces $\mathscr{G}_B$; Eq. (1b) enforces bilateral constraints; Eq. (1c) expresses the Signorini condition through a unilateral contact complementarity problem [6], in which $\phi(\mathbf{q})$ denotes the gap function and $\widehat{\gamma}_n$ the Lagrange multiplier of normal reaction; Eq. (1d) models 3D Coulomb friction in terms of the maximum dissipation principle [7, 8], resulting in a non-linear optimization problem; Eq. (1e) maps bodies velocity into the chosen coordinates derivatives. If we account for impulsive events (i.e. velocity discontinuities) and we reformulate both unilateral and bilateral constraints as conic complementarities, we can restate the DVI model (1) into a more compact Measure Differential Inclusion (MDI) form [9, 10, 11]

$$\mathbf{M}d\mathbf{v} = \mathbf{f}_t(\mathbf{q},\mathbf{v},t) + \mathbf{D}_\varepsilon d\boldsymbol{\gamma}_\varepsilon \tag{2a}$$

$$d\boldsymbol{\gamma}_\varepsilon \in \Upsilon_\varepsilon \perp \bar{\mathbf{u}}_\varepsilon \in \Upsilon_\varepsilon^* \tag{2b}$$

$$\dot{\mathbf{q}} = \Gamma(\mathbf{q})\mathbf{v} \tag{2c}$$

where $\mathbf{f}_t(\mathbf{q},\mathbf{v},t)$ are the total external impulses, $\mathbf{D}_\varepsilon d\boldsymbol{\gamma}_\varepsilon$ represents the effect of both unilateral and bilateral constraint impulses, Υ_ε denotes the merging of bilateral and friction second-order Lorentz cones (being Υ_ε^* its dual cone), and $\bar{\mathbf{u}}$ is a term related to generalized contact velocity.

For general-purpose applications, the use of a cone-complementarity time integration scheme on the full problem (2) effectively describes the evolution of the system. However, many important engineering fields – such as automated assembling lines, food industry, granular material processing, bulk material transportation – share the need of conveying a large number of small parts along a given track. Though the process is conceptually simple, the simulation of belt conveyors, vibratory feeders, or feeding tunnels is often afflicted by high computational burden and the various cases are managed through separate approaches. In particular, very small time steps are needed to capture the high-frequency impacts of vibrating feeders or large deformation of rubber belts.

3 PROPOSED METHOD

To address the problem, we propose a new class of contact constraints that models, in a unified approach, both the drifting of parts in vibratory feeders – without the need of simulating their high-frequency motion – and material advancement on sliding conveyors. In the proposed formulation, contacts are handled as rigid interactions between moving parts and a stationary object, where a time-dependent term is added to the complementarity constraint to enforce the tangential motion of the parts. This model can be interpreted as a homogenization of the high-frequency contact phenomena that generate drifting; effects such as friction limits in sticking/sliding or collision restitution are therefore preserved.

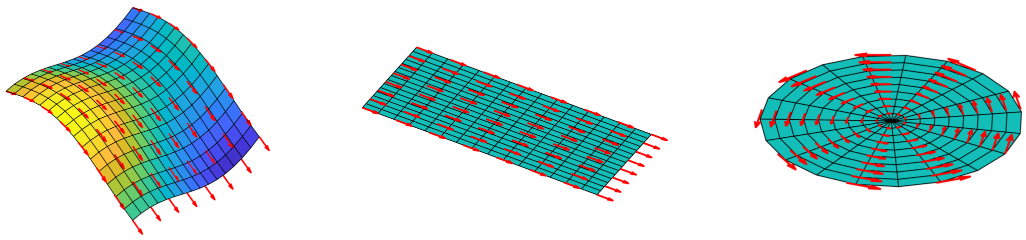

Figure 1: Illustration of proposed method, representing imposed contact velocity as red vector field. Left: generic surface with arbitrary motion law; center: linear sliding feeder, with constant unidirectional motion law; right: circular vibratory feeder, with tangential velocity increasing with radius.

Fig. 1 illustrates the overall concept. Desired eigenvalues of conveyor motion (being it a vibration or a continuous sliding) are provided by the user in the form of a velocity vector

$$\mathbf{v}_a = [v_x, v_y, v_z, \omega_x, \omega_y, \omega_z]^{\mathrm{T}} \tag{3}$$

expressed with respect to a convenient absolute reference frame. Given a generic track surface (imported from CAD or defined parametrically), a linear velocity is then calculated for each surface point:

$$\mathbf{v}_p = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} + \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} \times \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{4}$$

At this point, the frictional contact set-valued force laws [12] of MDI model (2) are extended to include a rheonomic term $\mathbf{c}_t = [c_{t_u}, c_{t_v}]^{\mathrm{T}} \in \mathbb{R}^2$. In particular, $(u, v, t) \mapsto \mathbf{c}_t$ is the mapping of $\mathbf{v}_p$ in the i-th $u-v$ contact plane, in which the normal velocity component n is discarded to avoid unnecessary bouncing of material on conveyor surface. If such speed map is known beforehand (e.g. from experiments or numerical/analytical models), it is possible to skip passages (3) and (4) and directly define $\mathbf{c}_t$; for example, a linear conveyor belt of constant speed s along the u direction would simply have $\mathbf{c}_t = [s, 0]^{\mathrm{T}}$.

In both cases, recalling notation of models (1) and (2), we may express each contact law at the i-th contact point as a rheonomic cone complementarity [13]

$$\gamma_i \in \Upsilon_{A,i} \perp \bar{\mathbf{u}}_i \in \Upsilon^*_{A,i}, \quad \forall i \in \{\mathscr{G}_A | \Phi_i = 0\} \tag{5}$$

where $\Upsilon_{A,i}$ and $\Upsilon^*_{A,i}$ are the i-th unilateral contact cones, ϕ_i is the contact gap and γ_i is the reaction impulse. The $\bar{\mathbf{u}}_i$ term is then restated in function of contact relative velocity $\mathbf{u}_i$, tangent velocity $\mathbf{v}_{\|,i} = \mathbf{u}_{u,i} + \mathbf{u}_{v,i}$, friction coefficient μ_i and the proposed $\mathbf{c}_t$ term:

$$\bar{\mathbf{u}}_i = \begin{bmatrix} u_{n,i} + \mu_i \|\mathbf{v}_{\|,i}\| \\ u_{u,i} + c_{t_u}(u, v, t) \\ u_{v,i} + c_{t_v}(u, v, t) \end{bmatrix} \tag{6}$$

With this modification, the high-performance and stability of the non-smooth formulation are preserved, while the simulation of transportation phenomena is performed through large time steps.

4 TESTING

In order to show the effectiveness of the proposed method, a number dedicated benchmarks have been implemented in C++ through the Chrono::Engine open-source library [14]. The tests have been computed on a Intel Core i7-10510U CPU, clocked at 1.80 GHz, with 4 physical cores and 16 GB of RAM.

Fig. 2 shows the example of a linear vibratory feeder for bulk material, composed by a vertical inlet hopper and an horizontal outlet trough. Transported parts – produced at a given flow rate – are represented by prismatic boxes whose sizes are randomly generated at runtime from a normal distribution; collisions, in this case, are computed between primitive shapes. At first, the system is simulated using a conventional approach: vibratory motion of the feeder is imposed through a dedicated motor, which is controlled in position with sinusoidal laws; upon impact, friction between trough and parts transmits motion to the latter. In a second run, the same system is simulated by the means of the proposed method. In this case, the feeder is modeled as a stationary object imposing a velocity law $\mathbf{v}_a = [v_x, 0, 0, 0, 0, 0]^{\mathrm{T}}$; proper rheonomic constraint $\mathbf{c}_t = [v_x, 0]^{\mathrm{T}}$ is applied to transported parts when contact occurs. In the bottom-right corner, Fig. 2 reports a comparison of material average drifting speed computed with the two approaches: while the overall behavior is consistent, the proposed formulation is able to proceed with a time step $h = 0.01$ s (Real Time Factor RTF $\equiv T_{\mathrm{cpu}}/T_{\mathrm{sim}} = 0.38$), while the conventional one requires $h = 0.001$ s (RTF $= 4.17$).

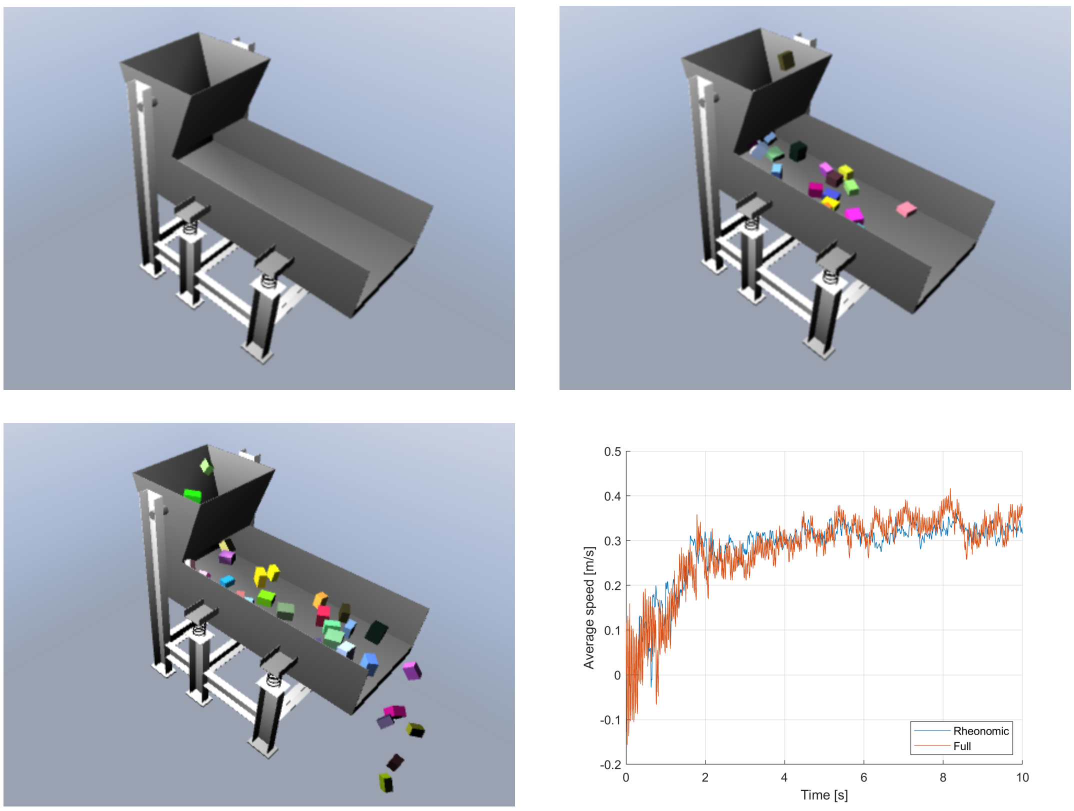

Figure 2: Simulation of a linear vibratory conveyor for bulk material transportation; parts have prismatic shape and normal distribution of size lengths. Bottom-right: plot of average material speed, comparing conventional full simulation approach (red) and proposed method (blue).

Fig. 3 illustrates a second benchmark, in which a vibratory bowl feeder is used to convey the same type mechanical components up to a helical track. At the beginning of the simulation, a given number of these parts are generated at random positions and orientations in the central area of the feeder. In this case, simulated body geometries are directly imported from a CAD software and their triangular mesh is used for both visualization and collision computation. The conventional approach models bowl vibration through a motor that produces screw oscillation about the vertical axis; the proposed method, on the other hand, imposes $\mathbf{v}_a = [0, v_y, 0, 0, \omega_y, 0]^{\mathrm{T}}$ and than internally

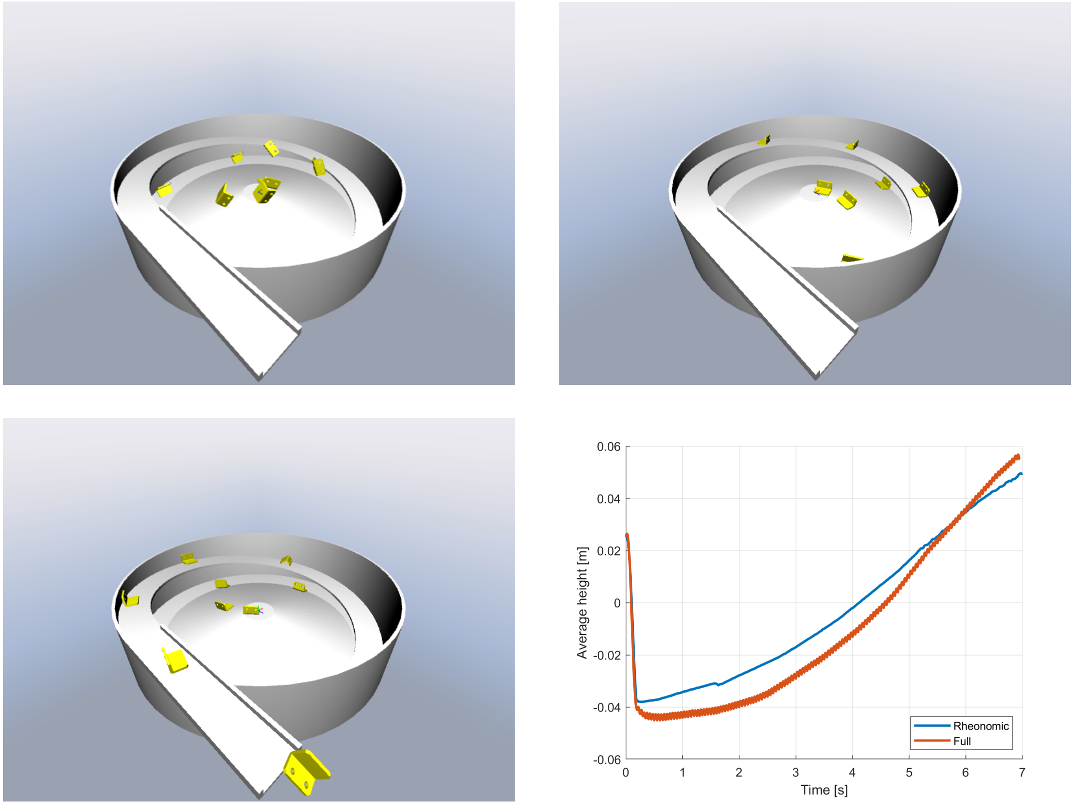

Figure 3: Simulation of a bowl vibratory conveyor for mechanical parts transportation along a helical track. Bottom right: plot of average parts vertical displacement, comparing conventional full simulation approach (red) and proposed method (blue).

computes proper $\mathbf{c}_t$ constraints through Eq. (4). The bottom-right part of the figure compares components average vertical displacement: after an initial drop (corresponding to parts falling on the feeder), components are progressively conveyed to the outlet chute. Again, the overall drifting phenomenon is preserved, but the proposed method evolves with a time step $h = 0.01$ s (RTF $= 2.05$) in contrast to the full simulation approach that requires $h = 0.005$ s (RTF $= 4.45$). It is worth noting that, in this example, the main computation bottleneck consists in collision detection among generic meshes.

5 CONCLUSIONS

We presented a non-smooth rheonomic constraint formulation that allows to efficiently simulate frictional contacts between generically-shaped parts and sliding or vibrating feeders. The reported benchmark tests show that the proposed method can be effectively used to simulate the overall material drift with a unified approach and with reduced computational burden.

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