Multibody joint parameter estimation using an Augmented Extended Kalman filter

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ABSTRACT

In recent years, state estimation techniques based on the Augmented Kalman filter have been successfully applied to nonlinear structural mechanics applications. These approaches typically assume an Ordinary Differential Equation description of the plant model used by the Kalman Filter. An Augmented Extended Kalman filter for Differential Algebraic Equation plant models, such as multibody models, has recently been introduced. With this method, the holonomic multibody constraints are enforced during the estimation via an additional projection onto the constraint manifold. This work proposes a generalization of this approach to include the estimation of holonomic constraint parameters, i.e. joint parameters. The estimation approach used in this work follows a three step procedure at each timestep: first, the state of the system is predicted by integrating the equations of motion, yielding an a-priori estimate; secondly, this estimate is corrected in order to decrease the mismatch between prediction and measurements, yielding an a-posteriori estimate; at last, this corrected estimate is enforced to lie onto the constraint manifold to ensure constraint satisfaction. As numerical validation of the methodology, estimation of angular misalignment for a revolute joint rotational axis is performed using simulated noisy measurements. The proposed estimator is able to successfully estimate the misalignment angle.

Keywords: Virtual Sensing, Kalman filter, Digital Twin, Constraints.

1 INTRODUCTION

In the context of Kalman filter-based estimation [1] for structural mechanics, this approach has been successfully applied in the field of input estimation [2, 3, 4], with some additional contribution related to parameter estimation [5]. In all of these works, the framework assumption in modelling approaches is the usage of Ordinary Differential Equations (ODEs). Recently, a new Kalman-based framework capable of dealing with Differential Algebraic Equations (DAEs) has been introduced [6, 7]. This approach has been used for input and force-element parameters estimation on multibody systems described by DAEs formulations. This work proposes a generalization of this approach for the estimation of parameters that influence the holonomic constraints expressions that characterize a multibody system. Examples of these parameters may be the location of the origin of a bracket joint on a specific body, or the orientation of a revolute joint's rotational axis with respect to a reference configuration.

Section 2 describes the theoretical material required for understanding the different aspects of this contribution. Section 3 is about the extensions made to the estimator in order to perform joint parameter estimation. Section 4 shows a numerical validation of the approach through an academic example in which the estimator is used to accurately estimate joint parameters. Conclusions and ideas for future applications of the developed methodology are presented in Section 5.

2 THEORETICAL BACKGROUND

2.1 Multibody formulation

The multibody formulation used in this work is briefly described in this section. The position and orientation $\mathbf{q}_i \in \mathbb{R}^{n_q}$ of a body i in space are defined in Cartesian coordinates as follows:

$$\mathbf{q}_i = \begin{bmatrix} \mathbf{q}_{r_i} \\ \mathbf{q}_{\theta_i} \\ \mathbf{q}_{f_i} \end{bmatrix} \tag{1}$$

where $\mathbf{q}_{r_i} \in \mathbb{R}^3$ is the vector of the X,Y,Z global position of the body reference frame, $\mathbf{q}_{\theta_i} \in \mathbb{R}^4$ is the vector containing the rotation coordinates in the form of quaternions, and $\mathbf{q}_{f_i} \in \mathbb{R}^{n_f}$ is the vector of n_f flexible coordinates. Its first derivative with respect to time completes the states for a single (flexible) body in space:

$$\mathbf{v}_i = \begin{bmatrix} \mathbf{v}_{r_i} \\ \mathbf{v}_{\theta_i} \\ \mathbf{v}_{f_i} \end{bmatrix} \tag{2}$$

Given the above generalized coordinates, the constrained equations of motion of an assembled multibody system composed of n_b bodies can be written as the following set of index-2 DAEs as [8]:

$$\dot{\mathbf{q}} - \mathbf{v} + \mathbf{G}_q^T(\mathbf{q})\mu = \mathbf{0} \tag{3a}$$
$$\mathbf{M}(\mathbf{q})\dot{\mathbf{v}} - \mathbf{f_e}(\mathbf{q}) - \mathbf{f_{nl}}(\mathbf{q},\mathbf{v}) + \mathbf{G}_q^T(\mathbf{q})\lambda = \mathbf{0} \tag{3b}$$
$$\mathbf{G}_q(\mathbf{q})\mathbf{v} = \mathbf{0} \tag{3c}$$
$$\mathbf{g}(\mathbf{q}) = \mathbf{0} \tag{3d}$$

Where:

- $\mathbf{q},\mathbf{v} \in \mathbb{R}^{n_q}$ constitutes the set of generalized coordinates and first derivatives;
- $\mu,\lambda \in \mathbb{R}^{n_g}$ are the two sets of Lagrange multipliers for the position and velocity level constraints, respectively;
- $\mathbf{g}(\mathbf{q}) \in \mathbb{R}^{n_g}$ is the set of holonomic algebraic constraints, with $\mathbf{G}_q(\mathbf{q}) \in \mathbb{R}^{n_g \times n_q}$ its derivative with respect to $\mathbf{q}$;
- $\mathbf{f_e}(\mathbf{q}),\mathbf{f_{nl}}(\mathbf{q},\mathbf{v}) \in \mathbb{R}^{n_q}$ are the vectors of generalized external forces and non-linear forces which include internal, gyroscopic and Coriolis terms and any other type of modeled force acting on the multibody systems.

In the context of input estimation with Kalman-based approaches, it is common to extend the set of index-2 DAEs (3) with a random walk model [9] to model the dynamics of the unknown parameters to be estimated. The resulting formulation becomes:

$$\dot{\mathbf{q}} - \mathbf{v} + \mathbf{G}_q^T(\mathbf{q})\mu = \mathbf{0} \tag{4a}$$
$$\mathbf{M}(\mathbf{q},\mathbf{p})\dot{\mathbf{v}} - \mathbf{f_e}(\mathbf{q},\mathbf{p}) - \mathbf{f_{nl}}(\mathbf{q},\mathbf{v},\mathbf{p}) + \mathbf{G}_q^T(\mathbf{q})\lambda = \mathbf{0} \tag{4b}$$
$$\mathbf{G}_q(\mathbf{q})\mathbf{v} = \mathbf{0} \tag{4c}$$
$$\mathbf{g}(\mathbf{q}) = \mathbf{0} \tag{4d}$$
$$\dot{\mathbf{p}} = \mathbf{0} \tag{4e}$$

where $\mathbf{p} \in \mathbb{R}^{n_p}$ is the set of inputs to be estimated. The variables can be reorganized as:

$$\mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \mathbf{v} \\ \mathbf{p} \end{bmatrix} \qquad \gamma = \begin{bmatrix} \mu \\ \lambda \end{bmatrix} \qquad \xi = \begin{bmatrix} \mathbf{x} \\ \gamma \end{bmatrix} \tag{5}$$

where $\mathbf{x} \in \mathbb{R}^{2n_q+n_p}$ is the augmented state vector, $\gamma \in \mathbb{R}^{2n_g}$ is the vector of Lagrangian multipliers, and $\xi \in \mathbb{R}^{2n_q+n_p+2n_g}$ is the total state vector. The augmented state $\mathbf{x}$ can be time-discretized by means of applying e.g. the first order Backward Differentiation Formula (BDF) time integration scheme, yielding:

$$\dot{\mathbf{x}}_{k+1} = \frac{\alpha_0}{h}\mathbf{x}_{k+1} + \frac{\alpha_1}{h}\mathbf{x}_k \tag{6}$$

in which h is the time step size and α_0 and α_1 are the BDF parameters. Substituting Eq. (6) in the set of augmented index-2 DAEs (4) and evaluating at the discrete time step $k+1$, the time-discretized, augmented set of index-2 DAEs becomes:

$$\mathbf{f_d}(\dot{\mathbf{x}}_{k+1}, \mathbf{x}_{k+1}) = \mathbf{f_d}(\mathbf{x}_{k+1}, \mathbf{x}_k) = \mathbf{0} \tag{7}$$

2.2 The Augmented Manifold Differential-Algebraic Extended Kalman Filter

The estimation framework used in this work is the Augmented MANifold Differential-Algebraic Extended Kalman Filter [6], or in short AMANDA-EKF. It is based on the well known Extended Kalman Filter (EKF) formulation [1]. In general, a nonlinear set of discretized ordinary differential equations (ODEs) can be written as:

$$\mathbf{f_d}(\mathbf{x}_{k+1}, \mathbf{x}_k) + \omega_{k+1} = \mathbf{0} \tag{8}$$

$$\mathbf{y}_{k+1} = \mathbf{h}(\mathbf{x}_{k+1}) + \varepsilon_{k+1} \tag{9}$$

where the function $\mathbf{h} \in \mathbb{R}^{n_y}$ transforms the augmented state $\mathbf{x}_{k+1}$ into the set of measurements $\mathbf{y}_{k+1}$, defining the so-called measurements equation. The terms ω, ε are zero-mean Gaussian noise terms with known covariance matrices $\mathbf{Q}, \mathbf{R}$, which represent unknown/unmodeled system dynamics (plant noise) and measurement noise, respectively.

The estimator provides an estimate of the first two statistical moments of $\mathbf{x}$ given the sequence of measurements $\{\mathbf{y_k}\}$. The first moment is the mean value of the estimate, referred to as $\hat{\mathbf{x}}$, the second moment is the state error covariance matrix $\mathbf{P}$.

An estimation timestep consists of a prediction step, in which the evolution of the estimate is predicted and the error covariance matrix is propagated using the state-space model of Eq. (4), and a correction step, in which the estimate is corrected as to match the provided measurement data. The prediction step is defined as:

$$\mathbf{f_d}(\hat{\mathbf{x}}^-_{\mathbf{k+1}}, \hat{\mathbf{x}}^+_k) = \mathbf{0} \tag{10a}$$

$$\mathbf{P}^-_{k+1} = \mathbf{F}_{k+1}\mathbf{P}^+_k\mathbf{F}^T_{k+1} + \mathbf{Q}_{k+1} \tag{10b}$$

And the correction step is defined as:

$$\mathbf{K}_{k+1} = \mathbf{P}^-_{k+1}\mathbf{H}^T_{k+1}(\mathbf{H}_{k+1}\mathbf{P}^-_{k+1}\mathbf{H}^T_{k+1} + \mathbf{R}_{k+1})^{-1} \tag{11a}$$

$$\hat{\mathbf{x}}^+_{k+1} = \hat{\mathbf{x}}^-_{k+1} + \mathbf{K}_{k+1}(\mathbf{y}_{k+1} - \mathbf{h}(\hat{\mathbf{x}}^-_{k+1})) \tag{11b}$$

$$\mathbf{P}^+_{k+1} = (\mathbf{I} - \mathbf{K}_{k+1}\mathbf{H}_{k+1})\mathbf{P}^-_{k+1} \tag{11c}$$

in which $\mathbf{F}$ is the transition matrix, $\mathbf{H}$ is the measurement Jacobian matrix, and $\mathbf{K}$ is the Kalman gain matrix .The superscript $\square^-$ refers to estimates or function evaluations corresponding to a prediction step (a-priori) estimate. The superscript $\square^-$ instead refers to estimates or function evaluations corresponding to a correction step estimate (a-posteriori). Throughout the estimation process, linearization of the discrete-time functions $\mathbf{f_d}$ and $\mathbf{h}$ is required to obtain the matrices $\mathbf{F}$ and $\mathbf{H}$, which can then be defined as:

$$\mathbf{F}_{k+1} = \left.\frac{d\mathbf{x}_{k+1}}{d\mathbf{x}_k}\right|_{\hat{\mathbf{x}}_k^+} \tag{12}$$

$$\mathbf{H}_k = \left.\frac{d\mathbf{h}(x_{k+1})}{d\mathbf{x}_{k+1}}\right|_{\hat{\mathbf{x}}_{k+1}^-} \tag{13}$$

The AMANDA-EKF requires the use and evaluation of the state transition matrix $\mathbf{F}$. Due to the nature of the set of augmented index-2 DAEs (4), calculation of this matrix when using the full state ξ would require the derivatives with respect to the Lagrange multipliers, typically resulting in non-trivial and complex operations. Therefore, as part of the AMANDA-EKF scheme, a projection approach is proposed to eliminate the Lagrange multipliers from the system state vector [6] and obtain $\mathbf{F}$ from derivation of the local state-space form of Eq. (4).

During the correction step the holonomic constraints are not enforced. The AMANDA-EKF formulation adds an additional step to the estimation process, in order to obtain a solution $\tilde{\mathbf{x}}_{k+1}^+$ that complies with the constraints and matches the estimated $\hat{\mathbf{x}}_{k+1}^+$ as close as possible. This is achieved by solving a constrained optimization problem that effectively enforces $\tilde{\mathbf{q}}_{k+1}^+$ to lie on the constraint manifold described by $\mathbf{g}(\tilde{\mathbf{q}}_{k+1}^+) = \mathbf{0}$. Using the final converged solution, the velocities can then be calculated as well. Finally, the constrained estimate $\tilde{\mathbf{x}}_{k+1}^+$ is assembled.

3 ESTIMATION OF JOINT PARAMETERS

A generalization of the AMANDA-EKF is proposed to also allow for joint parameter estimation. In the context of this work, a joint parameter is defined as a parameter related to the constraints that are present on the multibody system. This implies that the constraint function $\mathbf{g}$ is dependent from the joint parameter of choice:

$$\mathbf{g}(\mathbf{q},\mathbf{p}) = \mathbf{0} \tag{14}$$

With respect to Eq. (4), the presence of joint parameters implies that the formulation changes as following:

$$\dot{\mathbf{q}} - \mathbf{v} + \mathbf{G}_q^T(\mathbf{q},\mathbf{p})\mu = \mathbf{0} \tag{15a}$$

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{v}} - \mathbf{f_e}(\mathbf{q}) - \mathbf{f_{nl}}(\mathbf{q},\mathbf{v}) + \mathbf{G}_q^T(\mathbf{q},\mathbf{p})\lambda = \mathbf{0} \tag{15b}$$

$$\mathbf{G}_q(\mathbf{q},\mathbf{p})\mathbf{v} = \mathbf{0} \tag{15c}$$

$$\mathbf{g}(\mathbf{q},\mathbf{p}) = \mathbf{0} \tag{15d}$$

$$\dot{\mathbf{p}} = \mathbf{0} \tag{15e}$$

with the parameter vector $\mathbf{p}$ influencing $\mathbf{g}$ and $\mathbf{G}_q$. The approach used in Section 2.2 [6] can be modified to compute the additional derivatives with respect to $\mathbf{p}$ needed for evaluation of $\mathbf{F}_k$.

With reference to Section 2.2, the constraint enforcement step is also modified. Introducing the following vector:

$$\mathbf{z} = \begin{bmatrix} \mathbf{q} \\ \mathbf{p} \end{bmatrix} \tag{16}$$

A new constrained optimization problem is defined that enforces $\tilde{\mathbf{z}}_{k+1}^+$ to lie on the constraint manifold described by $\mathbf{g}(\tilde{\mathbf{z}}_{k+1}^+) = \mathbf{0}$. By solving this optimization problem, it's now possible to obtain the constraint compliant estimate $\tilde{\mathbf{z}}_{k+1}^+$. Using the final converged solution, the velocities can then be calculated as well. Finally, the constrained estimate $\tilde{\mathbf{x}}_{k+1}^+$ is assembled.

4 NUMERICAL VALIDATION

The proposed approach has been validated with a numerical example using the system in Figure 1. It consists of a rigid pendulum, connected with a revolute joint to the ground, which is represented by the frame (XYZ). The frame (xyz) is the revolute joint frame attached to the rigid body. This joint only allows for rotation of the body in a specific plane around the joint origin, and can be described by the following constraint equations:

$$g_1 = \angle(Z,z)_{XZ} = 0 \tag{17a}$$

$$g_2 = \angle(Z,z)_{YZ} = 0 \tag{17b}$$

The operator $\angle(a,b)_c$ defines the angle between axes a,b with respect to plane c. The net effect of constraints in Eq. (17) is that the rotational axis of the joint, which corresponds to axis z, is perpendicular to the XY plane. This particular system configuration is referred to as 'not misaligned'.

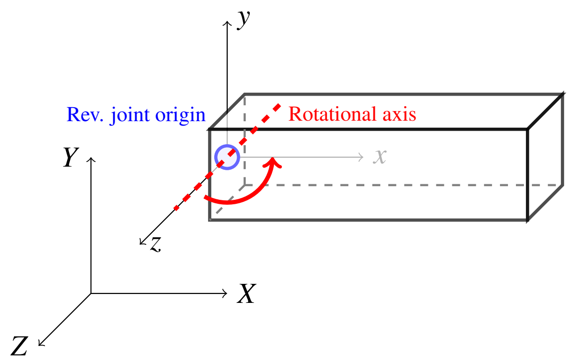


Figure 1: Visualization of the system in the reference configuration (not misaligned). The body's plane of rotation is the XY plane.

An additional configuration, shown in Figure 2, is used in this example. With respect to the previous configuration, shown in Figure 2 as well, the rotational axis of the revolute joint is no longer perpendicular to the XY plane. This configuration is referred to as 'misaligned'. The new rotational plane is now at an angle with respect to the XY plane. The corresponding constraint equations are modified as follows:

$$g_1 = \angle(Z,z')_{XZ} - p = 0 \tag{18a}$$

$$g_2 = \angle(Z,z')_{YZ} = 0 \tag{18b}$$

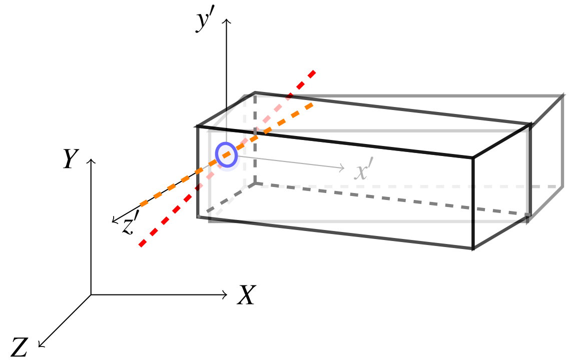


Figure 2: Visualization of the system in the misaligned configuration, compared to the reference configuration. The misaligned rotational axis is the dashed orange line, the body's plane of rotation is at an angle with respect to the XY plane.

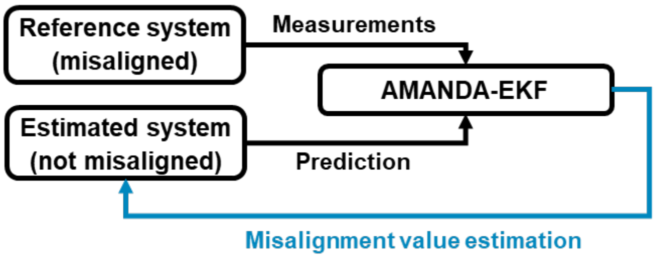


Figure 3: Concept scheme of the estimation workflow for the numerical validation.

The parameter p can, in this case, be referred to as the misalignment angle. In this example, this value is set to 5°. Having defined this two possible configurations for this system, the objective of this validation is accurate estimation of this parameter through the process represented in Figure 3:

- the misaligned configuration is used as reference model from which simulated noisy measurement signals are obtained;
- the not misaligned configuration is the system that is used in the AMANDA-EKF to obtain the predicted state of the system;
- both info are combined in the AMANDA-EKF to obtain a corrected estimate of the state of the system and of the misalignment parameter.

The measurement used for this example is the Z coordinate of the center of gravity of the misaligned pendulum. The noisy measurement is created by adding white noise with known covariance $R = 10^{-4}$ m^2 to the clean signal. $\mathbf{Q}$ is further defined as follows:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q_q} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q_v} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Q_p} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{n_q \times n_q} & \mathbf{0}_{n_q \times n_q} & \mathbf{0}_{n_q \times n_p} \\ \mathbf{0}_{n_q \times n_q} & \mathbf{0}_{n_q \times n_q} & \mathbf{0}_{n_q \times n_p} \\ \mathbf{0}_{n_p \times n_q} & \mathbf{0}_{n_p \times n_q} & 10^{-2}\mathrm{rad}^2 \end{bmatrix} \tag{19}$$

The inherent assumption in setting these values is that the main source of uncertainty between the reference and estimated model lies in the value of p. The filter will therefore reduce the mismatch between measured and predicted response by correcting the misalignment angle p.

The results of the estimation are shown in Figure 4 and 5. In particular, from Figure 4 it is possible to verify that the misalignment angle is correctly estimated to a reasonable degree of accuracy. In Figure 5a and 5b the estimation results regarding the position measurement signal are shown. For the not misaligned configuration, the sole prediction would dictate that the sensor readings would be equal to 0 for all time steps. However, due to the accurate estimation of the misalignment, it can be observed how the estimated signal tracks the reference signal, even in the presence of noise (Figure 5b).

5 CONCLUSIONS AND FUTURE WORK

A generalization of the AMANDA-EKF for the estimation of multibody joint parameter has been proposed in this work. The approach has been validated with a numerical example on an academic system, in which the angular misalignment of a revolute joint has been successfully estimated from simulated noisy measurements.

Future applications may include any where the uncertainty regarding the kinematics of the multibody system with respect to the real world counterpart needs to be reduced, since the methodology developed in this paper is applicable to a relatively wide class of problems. Future work will aim at applying the proposed methodology to the field of system level transmission simulation, allowing for the estimation and identification of drivetrain misalignments.

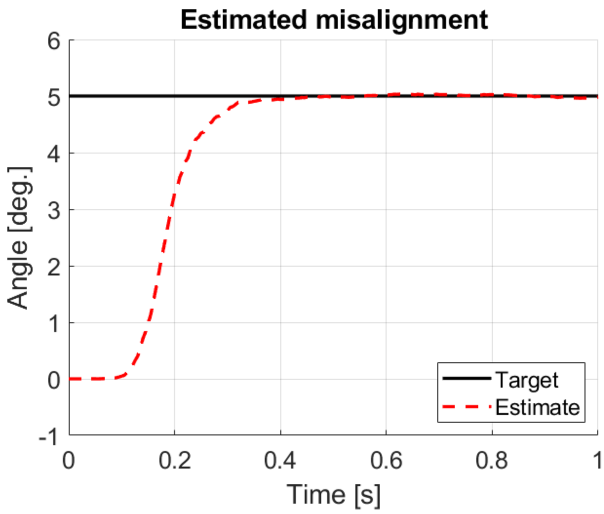
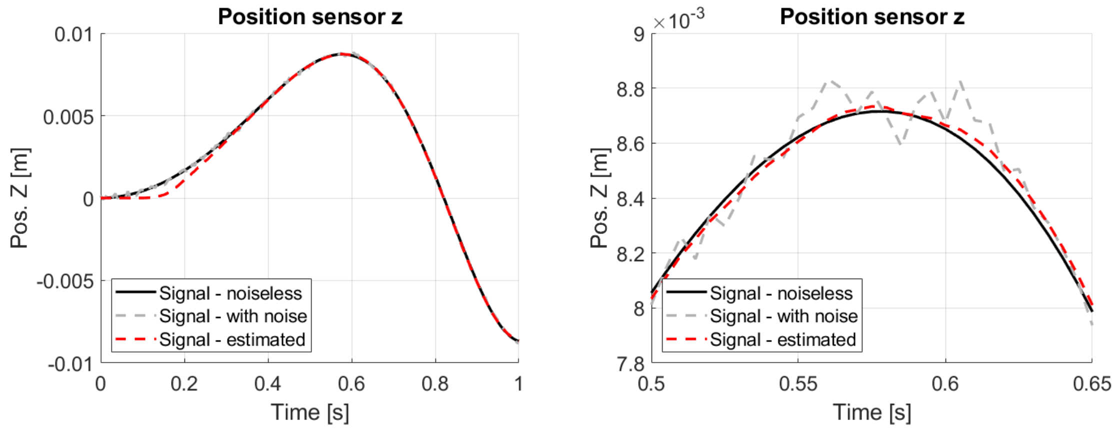


Figure 4: Angular misalignment estimate.



(a) Position sensor signal.

(b) Position sensor signal (zoom).

Figure 5: Measurement signal estimate.

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