Design Optimization of Multi-Elastic-Link Robots Based on Representative Load Cases

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ABSTRACT

Lightweight robots are playing an ever-increasing role in industrial processes. Therefore many control methods have been developed to handle occuring vibrations. This paper addresses the design optimization, based on the dynamic simulations for representative cyclic trajectories, instead. Serial manipulators with slender arms, which show significant structural elasticities, are considered. An elastic model on basis of a direct Ritz approach provides with model parameters which can be influenced directly by the construction (e. g. arm cross sections). Therefore the influence of design adjustments to the vibratory behaviour can be investigated trough simulations. This leads to a set of candidate designs. The stresses are used for a fatigue analysis and subsequent lifetime estimation of the critical structural elements. To this end a combination of the method of the critical cutting plane and the rainflow counting algorithm is utilized. The lifetime estimate is then used for choosing the final design out of a set of candidate configurations.

Keywords: elastic manipulator, Pareto optimization, fatigue analysis, structure analysis, structure oszillation.

1 INTRODUCTION

In industrial robotics, the requirements regarding time and energy optimality necessate the use lightweight robots. This is accompanied with several challenges. The major issue constitutes in the resulting structural elasticities which result in endeffector oscillations and therefore bad positioning accuracy. For the case of an already existing robot with no possibilities of design adjustments, one can use a variety of oscillation damping control methods as proposed in [1], [2] or [3]. Taking a step back to the design process of the robot itself, there are several possibilities to modify the structure in order to enhance its overall stiffness. A model of the robot, containing those targets as parameters, provides the engineer with the opportunity of assessing the effects of the changes made, via dynamical simulations. An intuitive and widely used method for modeling elastic multibody systems is the so called super element method, where the elasticities of whole substructures are taken account for by concentrated spring elements. The disadvantage of this method is, that the physical parameters of the robot are not directly related to the stiffness. The elastic model as shown in section 2, introduces the robots structural parameters dircetly as model parameters. The systematic variation of those parameters and the evaluation of specified quality criteria is the goal of the subsequent multicriterial optimization in section 3. Possible parameters as well as quality criteria and the used framework are introduced. As important the time-/energy-optimality and the handling aspects are, one also needs to assure a sufficient dimensioning in order to guarantee an appropriate lifetime of the robot. Section 4 of this paper is devoted to the question of how to properly estimate the lifetime of a structure under cyclic loading. Therefore some established methods of fatique analysis are combined to an algorithm which gets the simulated load data as well as the structural characteristics and provides the estimated lifetime as result. The whole methodology is then demonstrated in the subsequent section 5.

2 MODELING

This paper considers serial robotic manipulators consisting of a base with a rotational degree of freedom (DOF) around the vertical axis and one or more elastic arms with motor-gearbox units, which are identified as instances of a generalized syubsystem, see e. g. [4] for a formulation.

2.1 Gearbox elasticities

The gearbox with gear ratio i_G is modeled elastic with degrees of freedom q_M for the motor (the rotor velocity is then $\Omega_M = i_G \dot{q}_M$) and q_A for the link. The elastic transmission behavior is modeled as a coupling between q_A and q_M with the gear stiffness c.

2.2 Link elasticities

If the joint structure is assumed sufficiently stiff, the significant elasticities lie within the link. For a slender, homogenous arm with a constant cross section, the euler-bernoulli theory can be used. Elasticities in terms of multiaxial bending and torsion around the beam axis can be introduced as follows. Imagine a point $\xi \in [0, L]$ on the undeformed beam axis (L is the arm length), then $v(\xi,t) = \mathbf{v}(\xi)^T \mathbf{q}_v(t)$ and $w(\xi,t) = \mathbf{w}(\xi)^T \mathbf{q}_w(t)$ denote the time dependent displacement along the y-axis respectively z-axis. $\vartheta(\xi,t) = \boldsymbol{\vartheta}(\xi)^T \mathbf{q}_\vartheta(t)$ is the torsional angle around the x-axis. Vectors $\mathbf{v}(\xi)$, $\mathbf{w}(\xi)$ and $\boldsymbol{\vartheta}(\xi)$ denote the shape functions. Minimal coordinates $\mathbf{q}_e^T = \left(\mathbf{q}_v^T, \mathbf{q}_w^T, \mathbf{q}_\vartheta^T\right)$ are the time dependent Ritz coefficients. As the beam axis lies on the beam's centroidal axis, the vector to the center of gravity (COG) s of an infinitesimal beam slice and its orientation are given to

$$ {}_R\mathbf{r}_\mathbf{s}^T = \left(\xi \quad v(\xi,t) \quad w(\xi,t)\right)^T, \; {}_R\boldsymbol{\varphi}_\mathbf{s}^T = \left(\vartheta(\xi,t) \quad -w'(\xi,t) \quad v'(\xi,t)\right)^T. \tag{1} $$

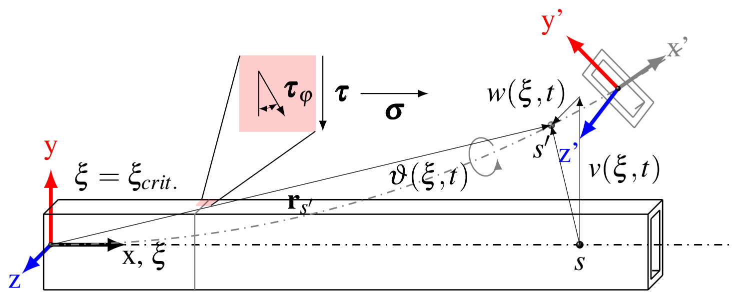

Figure 1. Elastic link (beam) of an articulated robot, with elastic displacements and critical point for the subsequent lifetime estimation (right)

The tendency of the link, to return to its undeformed shape is modelled by the reaction forces $\mathbf{Q}_{el} = -\mathbf{K}_{el}\mathbf{q}_e$, where the stiffness matrix is

$$ \mathbf{K}_{el} = \int_0^L \begin{bmatrix} GI_D \boldsymbol{\vartheta}'' \boldsymbol{\vartheta}''^T & 0 & 0 \\ 0 & EI_z \mathbf{v}'' \mathbf{v}''^T & 0 \\ 0 & 0 & EI_y \mathbf{w}'' \mathbf{w}''^T \end{bmatrix} dx, \tag{2} $$

with the Young's modulus E, the shear modulus G and the second moments of inertia I_y and I_z as well as the torsional moment of inertia I_D. For the goals of this paper, all important model parameters have been introduced at this point. For details on the modelling the reader is refered to [4].

3 Parameter Study

When it comes to the use of elastic lightweight robots, usually the reason is the desire for fast movements under low power consumption (e. g. pick and place tasks) and possibly restricted space. Therefore the primary design goal is the reduction of the moved robot mass. On the other hand, most tasks require a high positioning accuracy which means that oscillations, caused by structural elasticities, need to be sufficiently small. The goal to design the robot as light but also as stiff as possible, gives rise to the following multicriterial optimization problem

$$\min_{\mathbf{p}\in\mathbf{P}} \left(J_{\mathrm{m}}\left(\mathbf{p}\right), J_{\mathrm{osc.}}\left(\mathbf{p}, \mathbf{q}_d(t)\right)\right), \tag{3}$$

where $\mathbf{p}$ is a set of parameters and $\mathbf{P}$ denotes the set of valid parameters. $J_{\mathrm{m}}(\mathbf{p})$ assesses the configuration dependent robot weight, $J_{\mathrm{osc}}(\mathbf{p}, \mathbf{q}_d(t))$ denotes a quality criteria regarding the oscillation of the endeffector E and $\mathbf{q}_d(t)$ is the considered trajectory in the joint space.

3.1 Tuning Parameters

The parameters, easiest to vary, are the link geometries. The cross sections measurements affect the link mass as well as its stiffness (see eq.2). Especially when the links are already slender, a high proportion of the robots moved mass lies within the joints. As the joint structure might be quite complex it is varied by its mass and center of gravity (COG). However, a set of n parameters to be varied is denoted by $\mathbf{p} \in \mathbf{P} \subset \mathbb{R}^n$. Every set $\mathbf{p}$ leads to a robot configuration $c \in \mathbf{C} \subset \mathbb{N}$.

3.2 Quality Criteria

For measuring the robots quality regarding its mass the weight reduction in percent with respect to an initial robot configuration with mass m_0, $J_{\mathrm{m}}(\mathbf{p}) = \left(\frac{m(\mathbf{p})}{m_0} - 1\right) 100\%$, is used. Regarding the endeffector, one can focus on the position error $\Delta_I \boldsymbol{r}_E(t, \mathbf{p}) = {}_I\mathbf{r}_{E,d}(t) - {}_I\mathbf{r}_E(\mathbf{q}_M(t), \mathbf{q}_A(t), \mathbf{q}_e(t, \mathbf{p}))$ or/and the orientation error $\Delta_I \boldsymbol{\varphi}_E(t, \mathbf{p}) = {}_I\boldsymbol{\varphi}_{E,d}(t) - {}_I\boldsymbol{\varphi}_E(\mathbf{q}_M(t), \mathbf{q}_A(t), \mathbf{q}_e(t, \mathbf{p}))$. The vectors ${}_I\mathbf{r}_{E,d}(t)$ and ${}_I\boldsymbol{\varphi}_{E,d}(t)$ denote the desired position/orientation. Therewith several quality criteria as the errors whole time evolution or the maximum error $J_{\mathrm{osc.}} = \max({}_I\mathbf{r}_E)$ can be defined.

3.3 Load Case Simulation and Determination of Pareto Fronts

Determining the pareto efficient parameter set $\hat{\mathbf{P}} \subset \mathbf{P}$ requires the corresponding robot's response to a task representing load case in form of one or several trajectories. The determination of such a trajectory splits into two subtasks, see e. g. [5] for details on time-optimal trajectory planning for flexible link robots. First, the geometric path needs to be defined. The path velocity, acceleration and jerk depend on their limits which are implicitely set by the robot itself. Assuming an already defined, parametrized path $\mathbf{q}_d(\sigma)$ in the joint space, where $\sigma(t)$ is the path parameter, minimizing some cost function (e. g. the end time of the movement, the required actuation torque, ...) leads to the trajectory $\mathbf{q}_d(t)$. This optimization might be restricted by several constraints of the form

$$(.)_{min} \leq (.) \leq (.)_{max}, \tag{4}$$

where $(.)$ could be the velocity/acceleration in the joints, the velocity/acceleration of the endeffector or the available motor torque. In the context of this work it is most important to keep in mind that the velocity and torque limits depend on the used motor while the acceleration limits are determined by the torques and the masses to be moved. Therefore an initial robot configuration is needed to be defined, for which the representative trajectory is obtained.

The required response of the robot to the defined trajectory is generated by simulating a detailed dynamic model with all significant elasticities (see section 2). Accounting for the fact that the required detailing leads to a high model complexity, the $\mathscr{O}(n)$-algorithm for multibody systems in subsystem representation, as stated in [4], is used to calculate the minimal accelerations $\ddot{\mathbf{q}}(t)$. The numerical integration requires, due to the stiffness of the problem, an appropriate solver (e. g. ode15s). A central question is, how to calculate the actuation torques in order to track the desired trajectory.

There are many different approaches to the control of elastic systems, which aim in surpressing undesired oscillations. In our case, the goal is to design the robot towards good vibratory behaviour under the worst circumstances, therefore a simple PD joint control is used.

Plotting the evaluted quality criteria J_{osc} over J_{m} for every $\mathbf{p} \in \mathbf{P}$ leads to a Pareto-Front (see e. g. Fig. 4) which provides the basis for determining candidate configurations for a subsequent lifetime estimation.

The functionality of the developed framework shall be illustrated by the following pseudocode.

Algorithm 1: Multicriterial optimization procedure to determine the subset $\hat{\mathbf{P}} \subset \mathbf{P}$ of several possible robot configurations with respect to predefined quality criteria

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Input: Robot model, parameteres to be varied, quality criteria
Data: Representative trajectory $\mathbf{q}_d(t)$
for $\mathbf{p} \in \mathbf{P}$ **do**
  simulate the configuration c with parameters $\mathbf{p}$ for the representative trajectory $\mathbf{q}_d(t)$
  calculate the quality criteria
  save the configuration, simulation data and the values of the quality criteria
end
Result: results for every $\mathbf{p} \in \mathbf{P}$, Pareto-Front for every $c \in \mathbf{C}$

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4 LIFETIME ANALYSIS OF CANDIDATE DESIGNS

4.1 Fatigue Analysis

The cyclic loads, typical for a robot, might cause damage to the mechanical structure. Especially parts like the structurally weakened lightweight links are of interesst. Estimating this damage, accumulating it over time and calculating the resulting lifetime is the goal of fatigue analysis. Using simulation data, the estimated lifetime of a structural part is determined as

$$t_{\mathrm{life}} = \frac{t_{\mathrm{task}}}{D} \tag{5}$$

where $D \in [0,1]$ quantifies the damage which was caused due to the load over the time t_{task}. The load is the stress in the interesting part of the structure which leads to growth ΔD of the damage D and is given as a timeseries of normal and shear stresses. Damage growth ΔD is the result of a so called damage event, which is a constant amplitude σ_a stress reversal with mean stress σ_m. As the available material characteristics are always determined with regard to uniaxial test loads, one needs a hypothesis which maps the occurring normal and shear stresses to an equivalent normal stress. Such a hypothesis is refered to as equivalent stress hypothesis and for ductile materials, the maximum shear stress hypothesis (Tresca, see e. g. [6]) has been tried and tested. The time evolution of the equivalent stress needs to be searched for the mentioned damage events. A proven and widely used approach is the so called rainflow counting algorithm, see e. g. [6] or [7] for a modified version which accounts for load sequencing. The rainflow algorithm gets a load sequence and returns a discrete set of pairs (σ_m, σ_a) with their corresponding number of occurrences. The visualization of this data is usually refered to as *Rainflow-Matrix*. Every pair (σ_m, σ_a) is related to a damage growth ΔD_i.

The damage increases ΔD are accumulated to D according to a so called damage hypothesis. In this paper the Palmgren-Miner hypothesis

$$D = \sum_i \Delta D_i \tag{6}$$

is used. Miner's rule is a linear damage accumulation model that does not account for the load sequencing effect and nonlinear damage growth (as migth occur as a result of kinematic hardening), see e. g. [8].

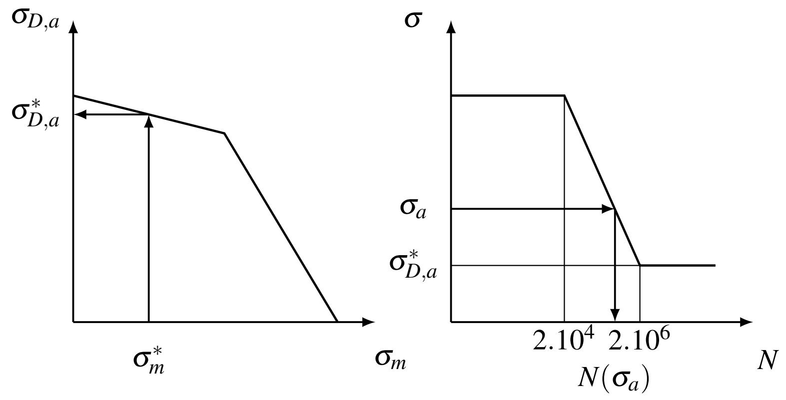

Figure 2. Haigh-Diagram and Wöhler-Diagram

4.2 Lifetime Estimation Procedure

As the Ritz coefficients $\mathbf{q}_e$ are accessible as a result of the dynamic simulation, it is possible to determine the elastic displacement of every point $\xi \in [0,L]$ on the beam axis. The corresponding normal and shear stresses are $\sigma(\xi,t)$ and $\tau(\xi,t)$. For homogenous beams with constant cross section, the weakest part of the mechanical structure tends to be the connection of the joints and the links, so this is the point of interest $\xi_{\text{crit.}}$. The proposed lifetime estimation, see algorithm 2, is based on a fusion of the method of the critical cutting plane and the rainflow method, with the cumulative damage equations according to Palmgren-Miner. For every possible cutting angle $\varphi \in \Phi$ the shear stress

$$\tau_\varphi(\xi,t) = -\frac{1}{2}\sigma(\xi,t)\sin(2\varphi) + \tau(\xi,t)\cos(2\varphi) \tag{7}$$

is calculated and used as input for the rainflow algorithm. For every pair (τ_m, τ_a) the corresponding pair (σ_m, σ_a), with $\sigma_m = 2\tau_m$ and $\sigma_a = 2\tau_a$ according to Tresca's hypothesis, is used to determine the related damage growth ΔD_i. The Haigh diagram Fig. 2 provides with the permanently bearable stress amplitude under a fixed mean stress. This $\sigma^*_{D,a}$ is then used to compute a synthetic Wöhler diagram which defines the bearable cycle counts $N(\sigma_a)$ for the specific σ_a. The damage growth is then calculated to

$$\Delta D = \frac{N(\tau_m, \tau_a)}{N(\sigma_a)}. \tag{8}$$

5 APPLICATION AND RESULTS

The following example shall illustrate the application of the methodology to an industrial robot. The section contains the definition of the parameters to be varied and the pareto optimization as well as the subsequent selection of the most promising configuration on basis of a lifetime estimation.

5.1 Parameter Study and Determination of Candidate Designs

For simplicity and to keep the example clear, only the wall thicknesses of the considered articulated robot's links are varied. Further variations might be cross sections measurements or even the used profile form. The reference robot's links have a quadratic cross section with edge length $a = 35\,\text{mm}$ and thickness $t_i = 4\,\text{mm}$. Varying t_1 (first link) and t_2 (second link) between 1 mm and 6 mm leads to 36 possible configurations. Writing them down as shown in Fig. 3 leads to the, as it shall be called in this paper, configuration matrix. Its entires correspond to those in the Parato-Front Fig.4.

Algorithm 2: Lifetime estimation of a critical point in the robot's structure on basis of the method of the *Critical Cutting Plane*. For converting the uniaxial loading sequence into an equivalent set of constant amplitude stress reversals, the *Rainflow Counting Algorithm* is used. The method assumes linear damage accumulation according to *Palmgren-Miner*.

Input: Set of cutting angles Φ, material specific Haigh diagram
Data: Normal and shear stresses $\sigma(\xi,t)$ and $\tau(\xi,t)$ at $\xi = \xi_{\text{crit.}}$
for $\varphi \in \Phi$ **do**
  calculate shear stress $\tau_\varphi(\xi,t) = -\frac{1}{2}\sigma(\xi,t)\sin(2\varphi) + \tau(\xi,t)\cos(2\varphi)$ for the φ cutting plane and $t = [0, t_{\text{task}}]$
  calculate rainflow matrix $\mathbf{M}(\tau_m, \tau_a)$ with the numbers $N(\tau_m, \tau_a)$ of reversals of the stress amplitudes with mean stress τ_m and stress amplitude τ_a
  for $(\tau_m^*, \tau_a^*) \in \mathbf{M}$ **do**
    begin calculate the related damage growth ΔD
      equivalent normal stresses $\sigma_m^* = 2\tau_m^*$ and $\sigma_a^* = 2\tau_a^*$ according to the maximum shear stress hypothesis
      fatigue strength $\sigma_{D,a}^*$ for σ_m^* from Haigh diagram
      damage $\Delta D = N(\tau_m^*, \tau_a^*)/N(\sigma_a)$ with $N(\sigma_a)$ from the synthetic Wöhler line for σ_m^*
    end
    $D(\varphi) := D(\varphi) + \Delta D$ according to the Palmgren-Miner damage hypothesis, $D \in [0,1]$ represents the damage, $D = 1$ means failure of the considered component
  end
  $D_{\max} := \max(D(\varphi), D_{\max})$
end
Result: Estimated lifetime $t_{\text{life}} := t_{\text{task}}/D_{\max}$

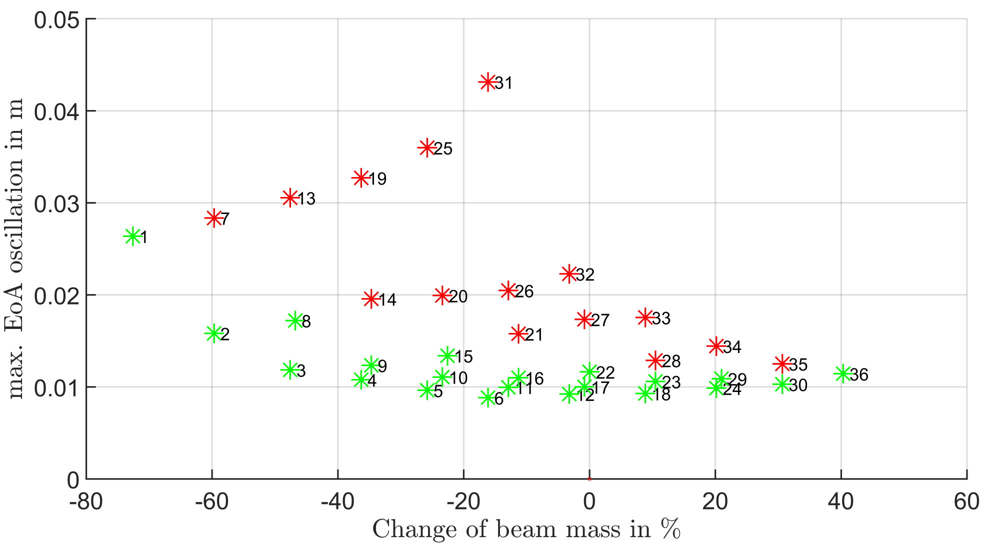

Figure 4. Paretofront: max. EoA-oszillation over mass reduction

As can be seen in Fig. 4 the configurations above the red line in the configuration matrix lie within a region of the pareto diagram which is quite uninteresting, also for the fact, that a second link thicker than the first one, does not make sense. Such configurations could be eliminated beforehand. One can identify groups of six configurations each (e. g. $1 - 7 - 13 - 19 - 25 - 31$, which lie on an imaginary line. Those groups represent configurations with a constant thickness

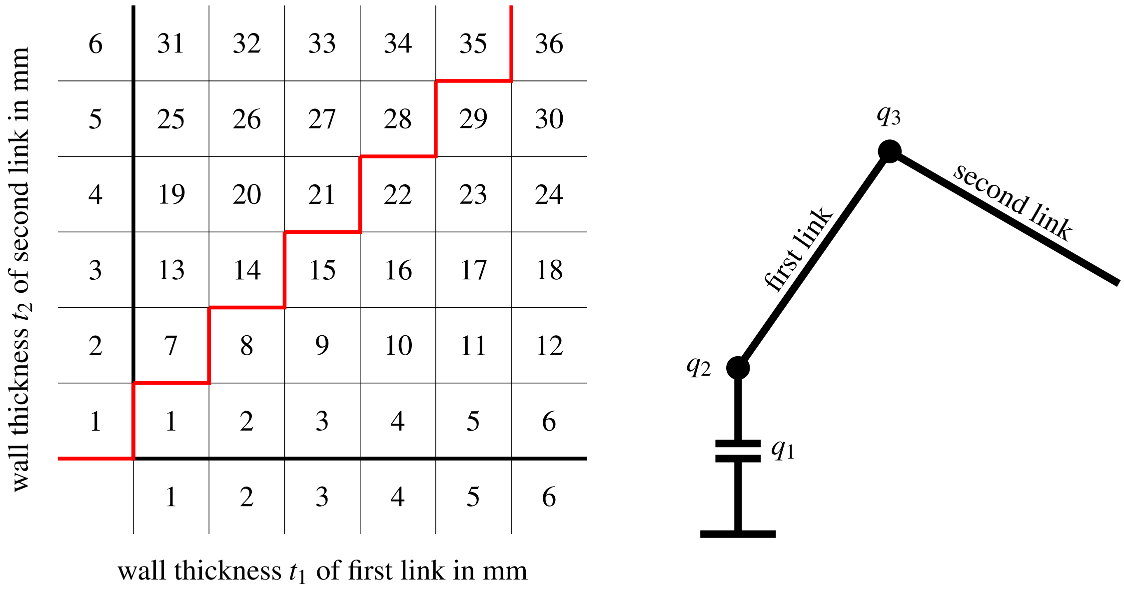

Figure 3. Configuration matrix for the chosen parameter ranges (left) and scheme of the articulated 3DOF elastic link robot (right)

of the first elastic link. They get steeply rising as this thickness decreases and the first part of the kinematic chain is weakened. As it is quite logical, the weight reduction should be obtained further ahead in the kinematic chain or at least equivalently distributed. As expected, the oscillations of the endeffector increase, as the links stiffness decreases with increasing mass reduction. On the other hand, with increasing mass, or more massive and therefore stiffer links, the oscillations also increase. This is due to the gearbox elasticities, which are excited as the weight of the arms gets higher and higher.

5.2 Fatigue Analysis of the Candidate Designs

As we are only interested in mass reduction, the pareto front in Fig. 4 defines a set $\hat{\mathbf{C}} = \{1,2,3,4,5,6\}$ of interesting configurations, which need to be further investigated in order to determine the final robot. The dynamical simulations of the configurations $\hat{c} \in \hat{\mathbf{C}}$ provide with the deformations of the elastic links and therefore the resulting stresses at a specific point. Applying algorithm 2 results in an estimated lifetime for each configuration, see Fig. 5. Some configurations do not reach stresses that go beyond the fatigue strength $\sigma^*_{D,a}$, therefore our specific approach does not yield any damage to the structure which result in an (theoretically) infinite lifetime. The corresponding configurations are marked with an upright triangle. From Fig. 4 and Fig. 5, we conclude that configuration $c = 3$ is the best one with respect to mass reduction, endeffector oscillation and estimated lifetime.

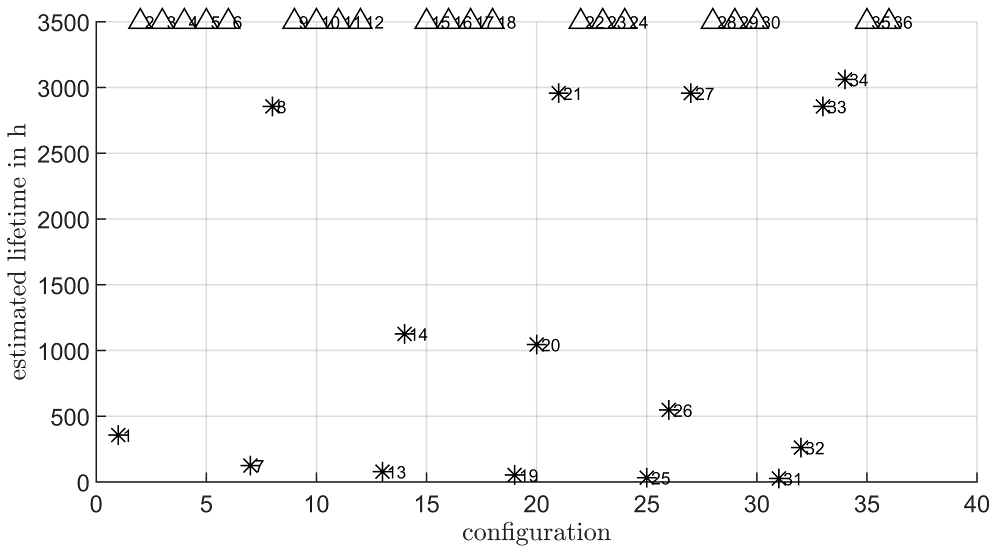

Figure 5. Estimated lifetime in hours for every $c \in \mathbf{C}$

6 CONCLUSIONS AND OUTLOOK

The introduced procedure for parameter tuning and lifetime estimation of elastic link robots provides a way to design and verify a lightweight robot with a (within the constructive possibilities) tailored dynamic behavior under assumption of a representative task trajectory. Applying it to an industrial pick and place robot, the multicriterial structure optimization resulted in a pareto front with respect to mass reduction and endeffector oscillation, where each point corresponds to parameters $\mathbf{p}$. The introduced lifetime estimation then provides a criterion for choosing the final design. An estimation of the remaining lifetime could be realized by utilizing the shown method for the damage calculation of the elastic links to accumulate the damage over the robots operation period to date. In addition to what we show in this work further weight reduction at the joints (drive units, etc.) might be quantified by additional elasticities, where the stiffness is determined by FEM simulations of the whole joint structure, as proposed in [9].

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