Tire cornering stiffness estimation using hybrid modeling

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ABSTRACT

Tires profoundly impact the dynamics of vehicles, influencing their handling, drivability, and ride comfort. Advanced chassis control systems used to improve vehicles' safety, performance, and reliability also require knowledge of tire behavior. Nevertheless, tires are very challenging to model as they are complex and non-linear components. Although simplified models are often employed, they are incapable of fully capturing tire behaviors. Using neural networks, i.e., black-box models, of the tire represents a common alternative. However, these approaches do not work outside the training data distribution, and they need costly and hard-to-measure experimental data for training purposes. Thus, this research study proposes a hybrid method while combining partially known physics of vehicle dynamics and a recurrent neural network to compensate for the unmodeled physics of tires. The developed approach learns the tire dynamics automatically from vehicle responses without requiring costly measured tire forces but solely relying on signals from an IMU. Lastly, the developed hybrid model is validated experimentally, providing accurate and stable results.

Keywords: Hybrid model, Tire forces, RNN, Track testing, Cornering stiffness

1 INTRODUCTION

With the advent of novel active safety systems and advanced driver assistant systems, it is of primary importance to accurately predict the states and behavior of the vehicle [1]. Tires play a key role in such dynamics of the vehicle, as they are the only mean for transferring forces and torques between the chassis and road, thus impacting its handling performance, comfort, and drivability [2]. Therefore, to control and correctly forecast the responses of a car, a sound knowledge of the tire and its properties is crucial. Directly measuring the forces developing on the tire involves sensors which are expensive and cumbersome to install, namely wheel force transducers. As a results, this does not represent a viable option for commercial vehicles or fleets. An alternative approach consists of precisely modeling the physical behavior of tire. Several tire simulation models are widespread in the field of vehicle dynamics, ranging from simplified linear models to the more complex non-linear ones, such as Dugoff, Pacejka and the Brush tire model [3]. In both cases, these models present some shortcomings. Concerning linear models are not able to capture the behavior of the tire, which is influenced by several factors like vertical loading, inflation pressure, temperature, and other operational conditions [4]. On the other hand, complex models like Pacejka' Magic Formula Tire model are capable to approximate such dynamics dependencies, but they require extensive testing to identify all their parameters [5]. During such routines, the tire is normally tested in lab conditions using costly test rigs, which might fail to reproduce the actual operating driving conditions.

Using data-driven black-box models is another possible solution to modeling tire dynamics. Thanks to the rapid growth and diffusion of Machine Learning and Deep Learning algorithms, during the last years, many works have focused on proposing methodologies based on Artificial Neural Networks for predicting tire forces and torques [6, 7, 8]. Even though such Deep Learning models can successfully approximate arbitrary non-linear and complex functions [9], they require large amount of costly experimental data to be trained in a Supervised Learning approach. Additionally, Neural Networks tends to suffer from a sharp drop in prediction accuracy when working outside the statistical distribution of the training data.

A third category of techniques for tire model estimation lies in filter-based methods [3, 10]. A wide range of estimators based on filtering algorithms have been developed over the years for hard-to-measure vehicle states. Most of these methodologies are based on different variations of the original Kalman filter [11]. An Extended Kalman filter which employs a random walk model for estimating an adaptive tire cornering stiffness was proposed in [12]. In [13], the authors opt for an Unscented Kalman filter, as it does not require the analytical computation of the Jacobian matrices. An Extended Kalman filter was also combined with a feedforward neural network, trained using high-fidelity simulation data, to predict tire lateral forces [14].

In this work, a hybrid method is proposed to estimate tire cornering stiffness and lateral forces. Such model is composed by both a white-box physical model and a black-box data-driven component, namely a recurrent neural network, which compensates for the unmodeled physics of the tires, to identify tire cornering stiffness and lateral forces. This approach learns such tire properties automatically according to the data collected from vehicle responses, without requiring costly measured tire forces but simply relying on signals from an inertial measuring unit and it can subsequently update itself against emerging changes due to environmental conditions, driving and operational. The hybrid model is trained using input data and available system states, along with an error defined on the output of the dynamic model. The model achieves good accuracy while overcoming some limitations of pure data-driven approaches and physics-based tire model.

The paper is structured as follows: in Section 2, the general framework of the proposed Hybrid model is described, while the choices of the physical model, Neural Network and Hybrid model architecture for the specific application of tire cornering stiffness estimation are detailed in Section 3. Lastly, the experimental validation of the proposed methodology and final considerations on the results achieved are outlined in the last sections.

2 HYBRID MODELING

In classical mechanics, physical concepts and analytical mechanics are used to describe the motion of macroscopic objects according to the pioneering formulations of Newtonian mechanics and later the reformulations of Lagrangian mechanics and Hamiltonian mechanics [15]. The resulting constitutive models are interpretable and generalizable. On the other hand, the complexity and nonlinearity of machines and multi-physics phenomena, environmental conditions, and a lack of information on how system parameters vary over time sometimes hinder the construction of efficient physical models or at least make it very difficult [16].

The recent development of sensing techniques and machine learning approaches have come into the scene to construct neural networks that can act as a black-box function to link inputs and outputs to predict motion of dynamic systems [17]. There are a wide variety of neural networks such as convolutional neural networks and recurrent neural networks, among others [18]. Recurrent neural networks (RNN) have a specific dynamic architecture giving the possibility to capture features of dynamics phenomena from time-series data [19]. These high-fidelity models suffer from the following drawbacks (*i*) interpretability; (*ii*) extrapolation; (*iii*) generalizability [20]. The physical relationships between inputs and outputs cannot be interpreted. The learning machines also are fundamentally interpolative and do not excel beyond the span of training dataset (the probability distribution). The generalizability also is essential as each dynamic system requires a new black-box setup and training, and the trained algorithms should be updated according to, for example, material degradation and aging, wear, and crack generation.

In data science, there is a possibility to integrate statistical learning concepts with classical approaches in applied mechanics and mathematics to discover sophisticated and accurate models of dynamical systems from time-series data. Schmidt and Lipson [21] used the genetic algorithm to discover motion equations from experimental data and Brunton et al. [22] developed a popular approach, the so-called SINDy. As the latter is limited to a function dictionary built by a user, a symbolic regression method to generate an adaptive function dictionary was suggested in [23]. These approaches have demonstrated a great capability in dynamical systems, fluid mechanics, and material science [23], but a lack of available data confines their success.

The cost of data acquisition is still prohibitive and there are critical locations of mechanisms inaccessible for instrumentation [24]. Subsequently, one is encountered with partial information from a physical identity, resulting in inaccurate training of neural networks. This issue can be alleviated with prior knowledge of a given system, e.g., physical laws and empirically validated formulations, which are not exploited in black-box models. Prior knowledge and information can guide the training process as a regularization agent such that efficient training with low amount of data becomes reachable. An example is such networks do not allow a solution that violates the mass conservation law in a mechanical system. Pioneering research work was published in [25] where Raissi and his colleagues nicely presented physics-informed neural networks.

The idea of combining neural networks and governing equations of a system can, in turn, be extended to deduce the structure of black-box networks for systems with incomplete known physics. A feasible challenge working with multibody systems is that one knows the physics of a given system partially due to associated complexity and nonlinearity, lack of information, and large multibody systems. Linking the known physics to data-driven models to compensate unknown physics can be helpful such that physical equations guide the training towards right solution quickly by confining the space of admissible solutions, reducing discrepancies between a completely known and incompletely known physics [16]. This approach called hybrid model improves predictions and mitigates the training issue of neural network with limited amount of data. In the hybrid models, neural networks are trained in a supervised manner using either direct or indirect measurements [16, 26]. This methodology also gives the possibility to identify system parameters [25]. Hybrid modeling is applicable to both rigid and flexibly multibody systems, Fig. 1., while the finite element method accounts for the elasticity of components [27]. Its applications encompass digital twins, system identification, condition monitoring, and design optimization.

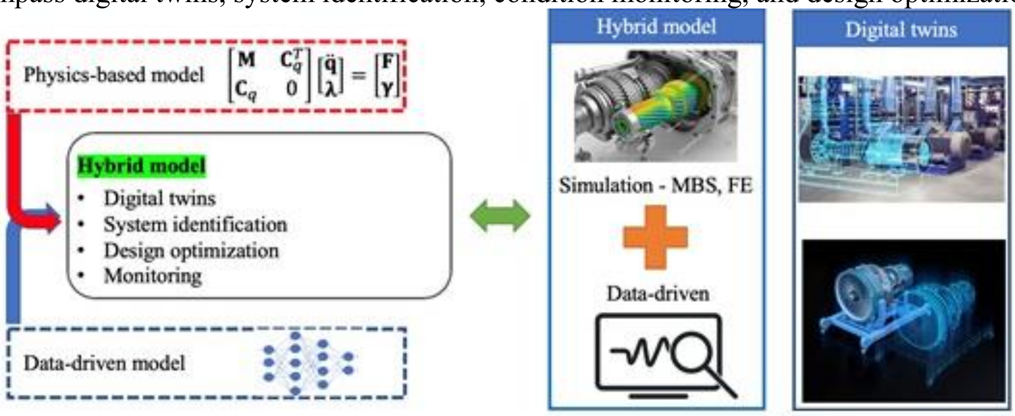

Figure 1. *Hybrid modeling framework and applications*

3 HYBRID MODELING FOR WHEEL LATERAL FORCE ESTIMATION

In this section, the hybrid model architecture for the estimation of lateral tire forces is introduced. After describing the physics model and its dynamic equations, the hybrid model is discussed. Finally, experimental dataset used for testing and validating the presented framework is outlined.

3.1 Vehicle dynamics

The multibody dynamics of a vehicle can be formulated by the equations of motion obtained from Newton-Euler equations to which constraint conditions are added as follows [28, 29]

$$\begin{bmatrix} \mathbf{M} & \mathbf{C}_q^T \\ \mathbf{C}_q & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \boldsymbol{\gamma} \end{bmatrix}, \quad \boldsymbol{\gamma} = -\left(\mathbf{C}_q \dot{\mathbf{q}}\right)_q - 2\mathbf{C}_{qt} - \mathbf{C}_{tt} \tag{1}$$

where **M** and **q** are the mass matrix and generalized coordinates of the system [30]. The force vector is designated by **F** that conveys external forces including normal and tangential contact forces [31, 32] along with Coriolis and centrifugal terms acting on the system bodies. **λ** and **C** are Lagrange multipliers and the holonomic algebraic constraints, respectively, and $\mathbf{C}_q$ is the derivative of the latter with respect to the coordinates. Employing a standard numerical integration technique, the above equation can be integrated over time [33]. For the chosen application, the motion of a vehicle can be described while the following assumptions are considered

- The model has only two degrees of freedom: yaw velocity and car body sideslip at the COG.
- Rolling and pitching motions are not considered.
- The aerodynamic efforts are neglected, so as the longitudinal forces.
- Angles are assumed small, and the trigonometric functions are linearized at the first order.

Given the previous hypothesis, the resulting dynamics equations, i.e., the bicycle model, are

$$\begin{cases} \dot{v_y} = \dfrac{1}{m}\left(F_{yf}\cos\delta + F_{yr}\right) - v_x r \\ \dot{r} = \dfrac{1}{I_{zz}}\left(l_f F_{yf}\cos\delta - l_r F_{yr}\right) \end{cases} \tag{2}$$

where v_x and v_y are the longitudinal and lateral velocities of the center of gravity (COG) of the vehicle, r is the yaw rate, F_{yf} and F_{yr} are the front and the rear tire lateral forces, m is the mass of the vehicle, I_{zz} is its yaw inertia, and l_f and l_r are, respectively, the distances of the COG from the front and rear axles. Assuming a linear tire model, the lateral forces can be expressed as

$$F_y = -C_\alpha \alpha, \tag{3}$$

where C_α is the two-fold cornering stiffness and $\boldsymbol{\alpha}$ the tire side slip angle. Given the small angles assumption, the side slip angles can be defined as

$$\alpha_f = \frac{v_y + l_f r}{v_x} - \delta, \qquad \alpha_r = \frac{v_y - l_r r}{v_x} \tag{4}$$

In such expressions, δ defines the steering angle acting on the front wheel. As the scope of this work is to estimate tire cornering stiffness without using expensive testing equipment, the chosen vehicle responses are the lateral acceleration a_y, the yaw rate r, and the vehicle side slip β, which can be easily measured and estimated using an inertial measurement unit (IMU). The latter vehicle response can be computed as

$$\beta = \tan^{-1}\left(\frac{v_y}{v_x}\right) \tag{5}$$

Accordingly, the bicycle model equations can be described in a state-space representation as

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}u \end{cases} \tag{6}$$

where

$$\mathbf{x} = [v_y, r], \; \mathbf{y} = [a_y, r, \beta], \; u = [\delta] \tag{7}$$

and

$$\mathbf{A} = -\begin{bmatrix} {(C_f + C_r)}/{m v_x} & V_x + {(l_f C_f - l_r C_r)}/{m v_x} \\ {(l_f C_f - l_r C_r)}/{I_{zz} v_x} & {(C_f l_f^2 + C_r l_r^2)}/{I_{zz} v_x} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \dfrac{C_f}{m} \\ \dfrac{l_f C_f}{I_{zz}} \end{bmatrix},$$
$$\mathbf{D} = \begin{bmatrix} \dfrac{C_f}{m} \\ 0 \\ 0 \end{bmatrix}, \qquad \mathbf{C} = -\begin{bmatrix} {(C_f + C_r)}/{m v_x} & {(l_f C_f - l_r C_r)}/{m v_x} \\ 0 & -1 \\ {1}/{v_x} & 0 \end{bmatrix} \tag{8}$$

The front and rear cornering stiffness, being the object of this work, are estimated by means of the black-box part of the hybrid model, i.e., a neural network, as explained in the next subsection.

From other point of view, one may eventually need to obtain wheel lateral forces. As such, one may use Eq. 2, consider cosine of that angle one due to low steering angle at tires, and finally the longitudinal velocity constant due to its slight changes over simulation time, leading to

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -v_x \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m} & \frac{1}{m} \\ \frac{l_f}{I_{zz}} & -\frac{l_r}{I_{zz}} \end{bmatrix} \begin{bmatrix} F_{yf} \\ F_{yr} \end{bmatrix} \rightarrow \dot{\mathbf{q}} = \mathbf{Aq} + \mathbf{Bu} \tag{9}$$

where

$$\mathbf{q}^T = [u_y \quad \theta_{yaw} \quad v_y \quad r], \;\; \mathbf{u}^T = [F_{yf} \quad F_{yr}] \tag{10}$$

and

$$\mathbf{y} = \begin{bmatrix} \dot{q}_3 + q_4 v_x \\ \dot{q}_4 \end{bmatrix} \tag{11}$$

From the state-space representation of system, the discrete form of the equation can be written as follows using the forward Euler integration formula.

$$\mathbf{q}_{k+1} = \mathbf{q}_k + (\mathbf{Aq} + \mathbf{Bu})_k \Delta t \tag{12}$$

in which k represent the state at time step k and Δt is the size of time step. The output of the physical system, i.e., $\mathbf{y}$, also includes the linear and angular accelerations of the automobile. Here, the time rate of yaw rate is obtained from either IMU or time-differentiation of yaw rate.

3.2 Neural Network and Hybrid Model Architecture

The architecture of the developed hybrid model is illustrated in Fig. 2 and is discussed in this section. The existence of inertia measuring unit (IMU) in vehicles gives the possibility to detect not only linear acceleration but also rotational rate using accelerometers and gyroscopes, respectively [34]. In the physical model of a vehicle, the force vector, $\mathbf{F}$, includes tire forces such as lateral loads, $\mathbf{F}_y$, which can assumably be related to sideslip angle, Eq. 3, linearly. Hence, one requires to estimate the cornering stiffness, C_α, to fully obtain corresponding loads. A neural network is combined to the physics-based model to account for the unmodeled physics of tire, identifying cornering stiffnesses of front and rear wheels, $\mathbf{z}$. Such a hybrid model does one step prediction, but one may be interested in a larger time horizon for which a RNN is used in here.

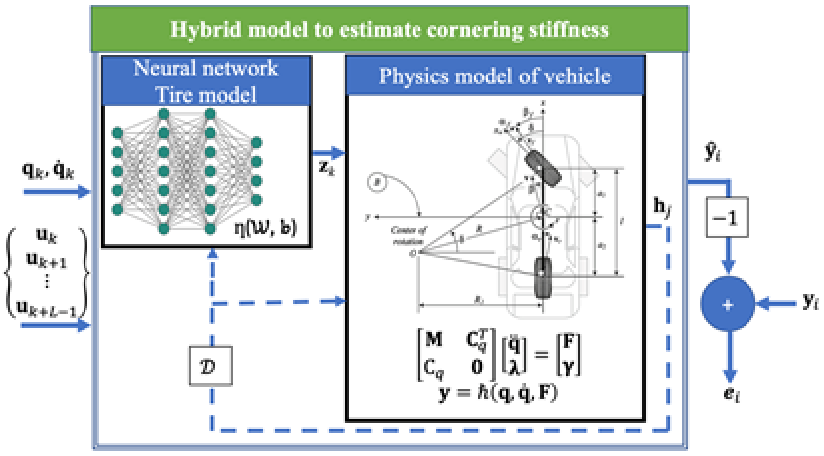

Figure 2*. Hybrid model to estimate lateral forces,* $\mathbf{z}_k$: $\{C\alpha_f, C\alpha_r\}$ *or* $\{F_{yf}, F_{yr}\}$, j: $k+1 \rightarrow k+L$, $\mathbf{y}_i = [y_k \quad y_{k+1} \quad \cdots \quad y_{k+L-1}]$, $\hat{\mathbf{y}}_i = [\hat{y}_k \quad \hat{y}_{k+1} \quad \cdots \quad \hat{y}_{k+L-1}]$.

This model is fed by (*i*) state of the system and (*ii*) input data of L sequence time steps. The corresponding outputs, $\hat{\mathbf{y}}$, are compared to those from measurement to estimate error, i.e., $\mathbf{e}$. the error is summed up over the whole-time window of experiment for constructing a loss function, Eq. 13, that is the root mean sum of square error (RMSSE). The training process aims at finding network parameters, i.e., weights and biases $\boldsymbol{\theta}$, along with possibly uncertain physical parameters of vehicle such that the loss function is optimized, Eq. (14).

$$\mathcal{L}(\theta) = \sqrt{\frac{1}{LN}\sum_{i=1}^{L-N}\sum_{k=i+1}^{i+N} \mathrm{e}_{k,i}^{T}(\theta)\mathrm{e}_{k,i}(\theta)}, \qquad \mathbf{e} = \mathbf{y} - \hat{\mathbf{y}} \tag{13}$$

and

$$\theta^* = \arg\min_{\theta}(\mathcal{L}(\theta)) \tag{14}$$

The back-propagation technique is used to correct network parameters, i.e., w_{ji}, to minimize the error in the system. In each iteration, a correction Δw_{ji} is applied to the synaptic weight w_{ji}, which can be obtained from the gradient of error with respect to corresponding network parameter, i.e., $\frac{d\mathcal{L}}{dw_{ji}} = \frac{d\mathcal{L}}{d\mathrm{e}}\frac{d\mathrm{e}}{dw_{ji}}$. To elaborate on the back-propagation algorithm, the technique is described for one output neuron in connection with the physics-based model, Fig. 3. The first term, i.e., $\frac{d\mathcal{L}}{d\mathrm{e}}$, can be calculated according to the loss function considered and the second term is obtained as follows

$$\frac{d\mathbf{e}}{dw_{ji}} = \frac{d\mathbf{e}}{d\mathbf{o}}\frac{d\mathbf{o}}{dz}\frac{dz}{dv_j}\frac{dv_j}{dw_{ji}} = -1\frac{d\mathbf{o}}{dz}\dot{\varphi}_j(v_j)x_i \tag{15}$$

in which the induced local field v_j is produced at the point of activation function φ_j, Eq. (16).

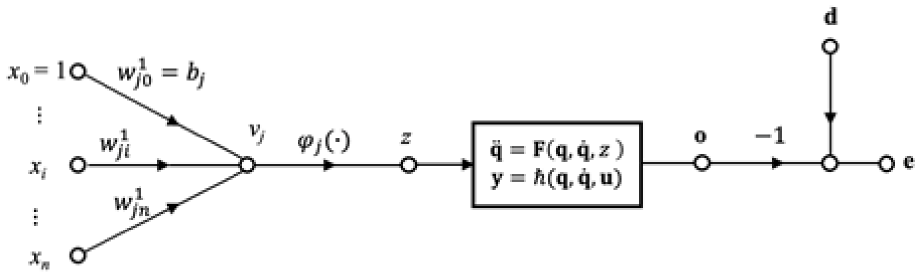

***Figure 3**. One signal-flow graph representation of neuron j in contact with physical model.*

$$v_j = \sum_{i=0}^{m} w_{ji}x_i \tag{16}$$

The differentiation of output, **o**, with respect to the neuron output, **z**, is done using three-points differentiation technique when the model is used to estimate cornering stiffness, otherwise the exact differentiation terms are calculated. According to the gradient descent procedure in weight space, the correction applied to w_{ji} is negative of loss function gradient with respect to weights and can be cast as

$$\Delta w_{ij} = -\eta\frac{d\mathcal{L}}{dw_{ji}} \tag{17}$$

where η is the learning-rate parameter. From a technical point of view, two RNNs are designed, in this study, each of which corresponds to each either cornering stiffness or lateral force, including four hidden layers with 25, 15, and 10 neurons. Linear and the rectified linear unit functions (ReLU) are also used as activation functions.

In what follows, the error estimation and back-propagation are described for the hybrid model to directly obtain lateral forces according to Eqs. 9-12. The error is estimated by comparing the augmented output of the multibody system with measurement as is given below

$$\mathbf{e}^{\boldsymbol{T}} = \mathbf{d}^{\boldsymbol{T}} - \mathbf{o}^{\boldsymbol{T}} = \begin{bmatrix} a_y & r & v_y & \dot{r} \end{bmatrix}_{exp} - \begin{bmatrix} a_y & r & v_y & \dot{r} \end{bmatrix}_{HM} \tag{18}$$

where **d** and **o** represent experimental data and those from the hybrid model. In order to calculate the gradient of the loss function with respect to network parameters, one requires to compute the alteration of error due to any differential changes in the input **z**, which is given as follows

$$\frac{d\mathbf{e}}{dz} = -1 \times \frac{d\mathbf{o}}{dz} \qquad z \in \{F_{yr}, F_{yf}\} \tag{19}$$

where

$$\frac{do}{dF_{yr}}^{T} = \begin{bmatrix} \frac{1}{m} & -\frac{l_r}{I_{zz}}dt & \frac{1}{m}dt & -\frac{l_r}{I_{zz}} \end{bmatrix} \qquad \frac{do}{dF_{yf}}^{T} = \begin{bmatrix} \frac{1}{m} & \frac{l_r}{I_{zz}}dt & \frac{1}{m}dt & \frac{l_r}{I_{zz}} \end{bmatrix} \tag{20}$$

3.3 Experimental dataset

The above proposed hybrid model is validated experimentally on a test vehicle. The chosen vehicle is an electric Siemens SimRod, in Fig. 4. In order to measure all the required signals, the SimRod has been equipped with an IMU, a wheel force transducer at each wheel and a potentiometer. The IMU allows to capture the accelerations and velocities of the car body along the XYZ axis as well as the corresponding angle rates (roll, pitch, and yaw). The Kistler wheel force transducers measure the forces and torques developing at the tire, which are needed as a reference to assess the accuracy of the predicted lateral forces. Lastly, the angle imposed by the driver at the steering wheel during track testing is acquired by means of the potentiometer, as it represents the input to the bicycle model, together with the longitudinal speed of the vehicle.

Figure 4. *Siemens SimRod electric vehicle, equipped with wheel force transducers.*

To thoroughly evaluate the lateral dynamics of the SimRod, different vehicle dynamics maneuvers are performed. Each maneuver is characterized by a different longitudinal speed as well as various profiles, amplitudes, and frequencies of steering wheel angle, as described in Fig. 5. In addition, overall mass of the SimRod is 930 Kg, its yaw inertia 700 Kgm^2, and the wheelbase length is 2.346 m. The distance of front axle to the COG is 1.175 m while steering ratio at wheels is 20.

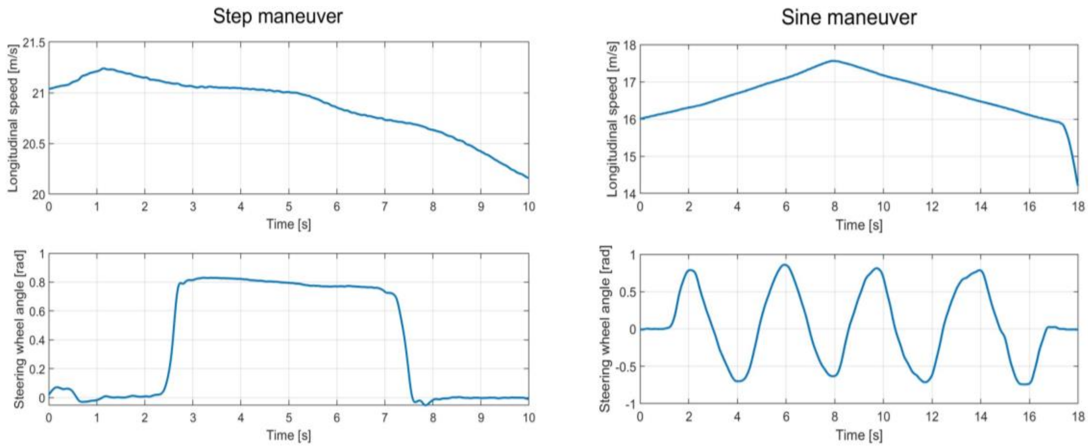

Figure 5. *Profiles of longitudinal speed and steering wheel angle for step and sine maneuvers.*

4 RESULTS AND DISCUSSION

This section aims at reporting on the validation and performance of the developed method. Before starting, it is worth mentioning that the programming codes to implement the presented method are all written in MATLAB (R2022b) and the algorithm is run on a 1.80 GHz personal computer with Intel(R) Core(TM) i5;8250U CPU. The prediction performance of the RNN hybrid model is evaluated by monitoring and analyzing the RMSSE. Moreover, the trained model is validated through the validation dataset that are sampled out from the track testing data in the first place.

The model first is examined and verified against a set of generated data from the available dynamics model of a vehicle while cornering stiffnesses values are considered constants. Then, the cornering stiffness of front and rear tires are estimated using experimental data from track testing of the SimRod. Specifically, a step maneuver is employed to check the developed model to directly determine cornering stiffness values of front and rear wheels. The learning process stops as the computation converges, Fig. 6d. The vehicle motion obtained using the estimated values of cornering stiffness of both tires, Figs. 6a and 6b, complies well with the measured data, namely lateral acceleration shown in Fig. 6c. The lateral forces are also calculated, Eq. (3), and good agreements were observed in comparison with those from measurements.

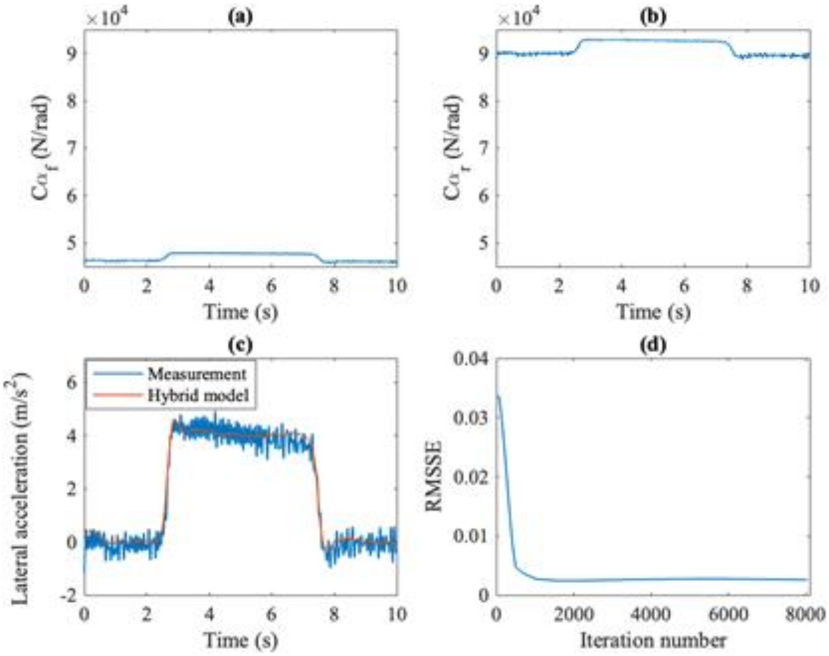

***Figure 6**. Estimating cornering stiffness: (a) front wheel cornering stiffness, (b) rear wheel cornering stiffness; (c) lateral acceleration; (d) root mean sum of squared error.*

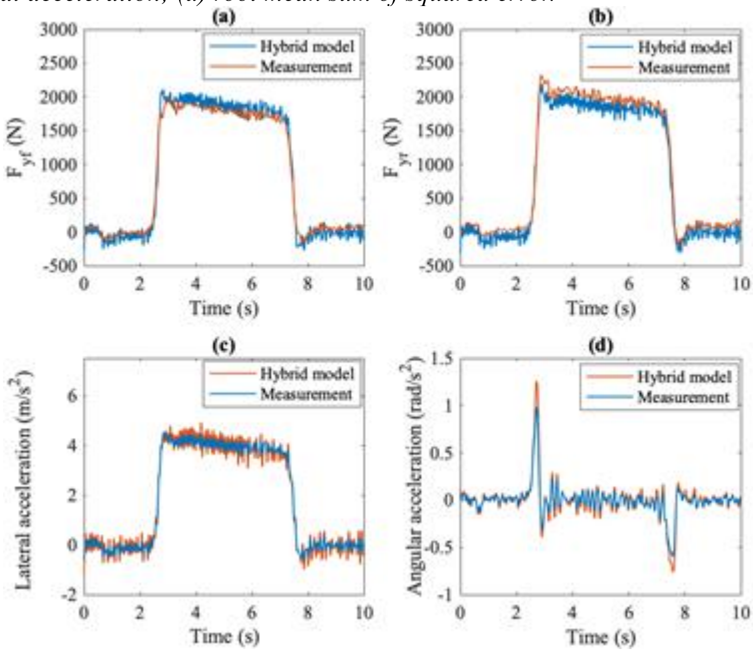

***Figure 7**. Estimating lateral forces from step maneuver data: (a) front wheel lateral forces, (b) rear wheel lateral forces; (c) lateral acceleration; (d) angular yaw acceleration.*

There is an observation issue of cornering stiffness magnitudes according to Eq. (3) that results in the matrices defined in Eq. (8), which is the cornering stiffness cannot be observed when sideslip angle is zero, Fig. 5. One may argue that the cornering stiffness values, reported in Fig. 6, associated with that time period are not reliable. A remedy is to estimate directly wheel lateral forces instead, as is formulated in Eqs. (9) – (12). Using such a model, respective results of two different maneuvers, namely step and sine, are acquired. The corresponding outcomes include lateral tire forces, which are compared to those measured by means of wheel force transducers and presented in Figs. 7a, b and Figs. 8a, b, and vehicle responses, i.e., linear acceleration and angular yaw rotations compared to experimental data in Figs. 7c, d and Figs. 8c, d, respectively. One can conclude that the outcomes comply well with measured data of the automobile motion and tire loads. It demonstrates the capability of developed approach to predict vehicle parameters and tire non-linear dynamics without costly and hard-to-install instrumentations. This model achieves good accuracy, while overcoming some of the limitations of pure data-driven approaches and physics-based tire model. The procedure is generalizable as can update itself against emerging changes due to environmental conditions, driving and operational as well as tire conditions, e.g., tire's inflation pressure, wear, and temperature. As the learning process becomes an inherent characteristic of this model while using the data measured by means of the vehicle sensors, one can utilize this hybrid simulation for different vehicle variants and tire dimensions.

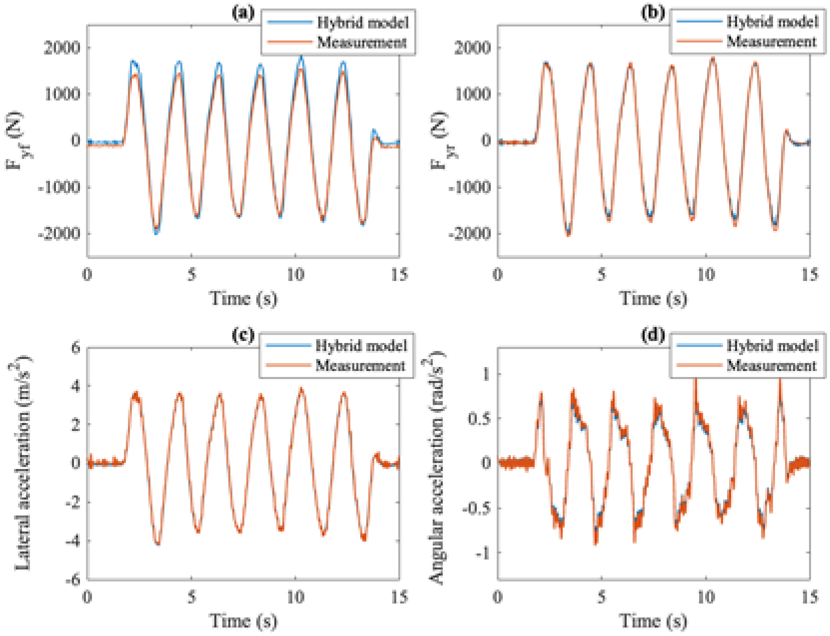

Figure 8. *Prediction of lateral forces from sine maneuver data: (a) front wheel lateral forces, (b) rear wheel lateral forces; (c) lateral acceleration; (d) angular yaw acceleration.*

5 CONCLUSIONS AND FUTURE WORK

A hybrid model was developed to predict tire cornering stiffness and lateral forces, linking a white-box physical vehicle model to a recurrent neural network. The latter compensated the unknown physics of tire behaviors being very complex and nonlinear. The suggested approach learnt the unknown sub-system of a multibody system automatically in a supervised manner using merely the vehicle responses collected by means of an IMU without any need of costly wheel force transducers to measure tire forces. The method was validated against experimental data for multiple maneuvers. One can conclude that the hybrid model complied well with measured data of the automobile motion. Overall, the capability of the developed method to estimate vehicle parameters and tire's non-linear dynamics without costly and hard-to-install instrumentations was demonstrated. This is an ongoing research study that aims at extending the developed model by integrating more sophisticated vehicle model of 15 DoF. The other research direction is to develop a robust and efficient differentiation technique outperforming available methods for noisy data.

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